Interconnectedness as a Source of Uncertainty for Systemic Risk*

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Today

- Methodology to compute the Probability of Systemic Default
  - Network context
  - Contracts and holdings
    - External Assets
    - Collateralized Loans

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  - Network context
  - Contracts and holdings
    - External Assets
    - Collateralized Loans
- Capacity of regulator to assess Systemic Risk in an **interconnected** system
  - Multiple Equilibria arise due to specific connectivity patterns
  - Uncertainty on
    - Probability of Systemic Default
    - Expected Losses

Motivation

Since the beginning of Great Recession

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- Regulators warning
  No satisfactory framework yet to deal with too-big-to-fail institutions and systemic events of distress in the financial system

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  1. balance sheet interlocks (e.g. credit, repo, derivatives, etc.)
  2. indirectly via exposures to common assets

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Challenge

Default Probability of one institution in a networked system.

(Greenwald, 2003), (Stiglitz, 2009), (Gai and Kapadia, 2010), (Cont et al., 2012), (Battiston et al., 2012), (Gourieroux et al., 2013), (Ota, 2014).

This work

- Contribution of this work
  1. Develop methodology to compute the default probabilities **ex-ante**
  2. Show conditions for **systemic risk uncertainty** in an interconnected financial systems
  3. Quantify the effects of **network structure**, **correlations**, **cyclicality**, **leverage** and **volatility**

This work

- Contribution of this work
  1. Develop methodology to compute the default probabilities **ex-ante**
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  3. Quantify the effects of **network structure**, **correlations**, **cyclicality**, **leverage** and **volatility**
- Policy Implications

  Large Uncertainty on Estimation of Systemic Risk

  1. Market structure
  2. Activity supervision and data collection
  3. Regulator intervention

The Model

- Builds on method à la (Eisenberg and Noe, 2001), (Cifuentes et al., 2005)
- Generic Approach (Gai et al., 2011), (Beale et al., 2011), (Arinaminpathy et al, 2012)
- Focus on Default Probability (Gourieroux et al., 2013), (Ota, 2014)

The Model

Time 1 Banks allocate assets and liabilities

Time 2 Shocks hit external assets, some banks may default and this affects counterparties

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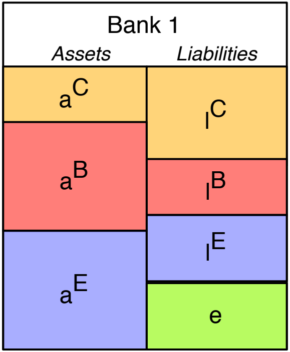
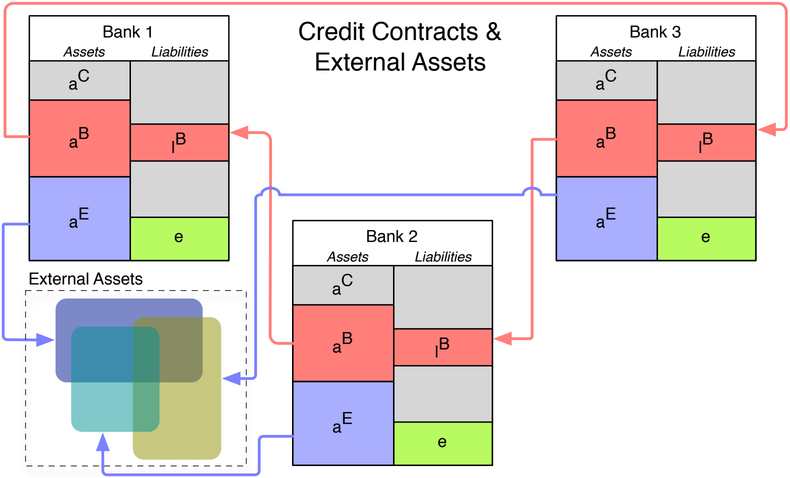
Balance Sheet



- Collateral
- Interbank Market
- External Markets

Interbank Credit Market



Model set-up

External assets at time 2

- $a_i^E(2) = a_i^E(1) \sum_k E_{ik} x_k^E(2) = a_i^E(1)(1 + \mu + \sigma\, u_i)$
  - μ_i: expected return
  - σ_i: standard deviation
  - u_i: a r.v. with mean 0 and variance 1
  - $p(u_1, ..., u_n)$: joint probability distribution of shocks

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Interbank assets at time 2

- $a_i^B(2) = a_i^B(1) \sum_j B_{ij} x_j^B(2)$
  - B_{ij}: fraction of i's interbank assets invested at time 1 in the liability of j
  - x_j^B: unitary value of j's interbank liability

$$x_j^B(1) = 1 \forall j \quad \text{and} \quad x_j^B(2) = \begin{cases} R \text{ if bank j default} \\ 1 \text{ else} \end{cases}$$

Model set-up

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Interbank assets at time 2

- $a_i^B(2) = a_i^B(1) \sum_j B_{ij} x_j^B(2)$

Collateralised assets at time 2 (risk-free assets)

- $a_i^C(2) = a_i^C(1) = \sum_j R_{ij} l_{ij}^B$
  - R_{ij}: fraction interbank liability l_{ij}^B secured by the collateral

Default condition

Negative Equity

$$
\begin{aligned}
e_i(2) =& a_i(2) - \ell_i < 0 \\
=& a_i^E(1)(1 + \mu + \sigma\, u_i) + a_i^B(1) \sum_j B_{ij} x_j^B(2) + a_i^C(1) - \ell_i < 0
\end{aligned}
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\end{aligned}
$$

Rewrite in relative terms: $e_i(2) < 0$ if $\frac{e_i(2)}{e_i(1)} < 0$

$$
\varepsilon_i(1+\mu+\sigma\, u_i) + \beta_i \sum_j B_{ij} x_j^B(2) + \gamma_i - \lambda_i < 0
$$

where

- ε leverage over external assets
- β leverage over (unsecured) interbank assets
- γ leverage over collateralised assets
- λ leverage (debt/equity), $\lambda_i = \varepsilon_i + \beta_i + \gamma_i - 1$

Default condition

Express default as a function of the external shock

$$u_i < \theta_i \equiv \frac{1}{\varepsilon_i \sigma}(-\varepsilon_i \, \mu + \beta_i(1 - \sum_j B_{ij} x_j^B(\chi_j) - 1))$$

where:

- χ_j is a default indicator

$$\chi_j = \begin{cases} 1 \text{ if bank } j \text{ default} \\ 0 \text{ else} \end{cases}$$

Extreme cases

1. Case no bank defaults $\theta_i = \theta_i^- = -\frac{1}{\varepsilon_i \sigma}(\varepsilon_i \, \mu + 1)$
2. Case all banks default $\theta_i = \theta_i^+ = -\frac{1}{\varepsilon_i \sigma}(\varepsilon_i \, \mu - \beta_i(1 - R) + 1)$

Equation System

For a given combination of shocks $u = \{u_1, ..., u_n\}$

$$\forall i \quad \chi_i = \Theta(\theta_i(\chi_1, ..., \chi_n) - u_i),$$

where

- Θ is a Heaviside function (step function)

A solution of the system above is denoted as χ^* (**Equilibrium**)

Default Probability

Individual Default Probability of bank i, P_i

$$\forall i \quad P_i = \int \chi_i^*(u)\, p(u)\, du$$

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Systemic default probability P^{sys}

$$P^{sys} = \int \chi^{sys}(u)\, p(u)\, du$$
$$= \int \Pi_i \chi_i^*(u)\, p(u)\, du$$

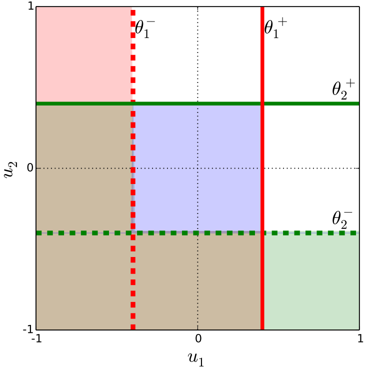
(Example)

with $p(u)$ joint density function of shocks

Simple Example

System of 2 banks lending and borrowing form each other

2-Dimensional State Space



$$\theta_i = \begin{cases} \theta_i^- & \text{when } j \text{ does not default} \\ \theta_i^+ & \text{when } j \text{ defaults} \end{cases}$$

Results: Multiple Equilibria

Proposition: Multiple Equilibria

Consider the case of N banks, with: recovery rate $R_i < 1$; interbank leverage $\beta_i > 0$; external leverage ε_i and shock variance σ_i positive and finite; shock mean μ finite.

Multiple equilibria exist if and only if:

1. there exists a **cycle** C_k of credit contracts along $k \geq 2$ banks
2. for each bank i and its borrowing counterparty $i+1$ along the cycle C_k, it holds $\hat{\theta}_i(\chi_{i+1} = 0) \neq \hat{\theta}_i(\chi_{i+1} = 1)$

where $\hat{\theta}_i = min\{max\{\theta_i, -1\}, 1\}$

Results: Multiple Equilibria

Figure: Example of network structures

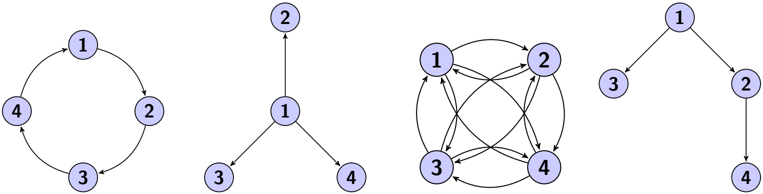


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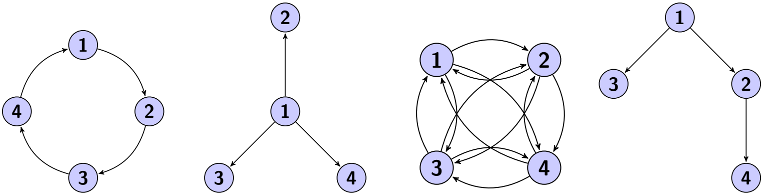
Figure: Example of network structures



Corollary

An interbank market where banks only act as **borrowers** *or* **lenders** *always lead to a* **unique equilibrium** *for the default state.*

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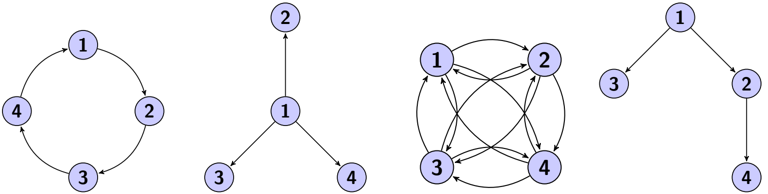

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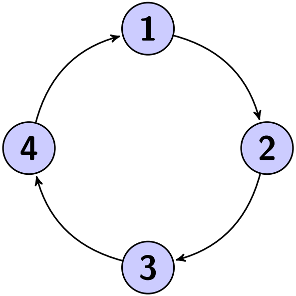
Note: Many real world financial networks exhibits many cycles (e.g. core-periphery structures (Craig and von Peter, 2014))

Case Study: Ring Market

Proposition: Uncertainty along one Cycle

$$\Delta P = \Pi_i^n \left(\frac{\beta_i (1 - R_i)}{2 \varepsilon_i \sigma_i} \right)$$



- $\uparrow$ with **interbank leverage**
- $\downarrow$ with **fraction of collateral**
- $\downarrow$ with **external asset leverage**
- $\downarrow$ with **variance on ext. shocks**
- $\downarrow$ with **length**

Discussion

- Mathematically: default state condition lead to multiple solutions
- Economically:
  - We can think they refer to different beliefs in the default of others and assume a prior
  - There is no way ex-ante to select a solution without introducing further assumptions.

Examples:

- 2012 Draghi's statement: "We will do whatever it takes"
- Moral hazard debate

Conclusions

- Investigate effect of network structure on capacity of regulator to assess systemic risk
- New methodology to compute analytically the default probabilities of n banks in a network of contracts
- Multiple equilibria arise even with only "mechanistic" properties
- Uncertainty on systemic risk level due to network properties: cycles
- Show the interplay between uncertainty and leverage, volatility, correlations and network properties
- Implications for analysis quality and intervention decisions

Thank You!

Uncertainty Probability of Systemic Risk

Multiple Equilibria imply multiple solutions for P^{sys}

$$\rightarrow \text{multiple vectors } \{\chi_1^*, \chi_2^*, .., \chi_n^*\}$$

Let us focus on the extreme cases:

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- $P^+ = \int \chi_{sys}^+(u)p(u)d(u)$ $\rightarrow$ Under **optimistic** scenario

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- $P^- = \int \chi_{sys}^-(u)p(u)d(u)$ $\rightarrow$ Under **pessimistic** scenario

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Let us focus on the extreme cases:

- $P^+ = \int \chi_{sys}^+(u)p(u)d(u)$ $\rightarrow$ Under **optimistic** scenario
- $P^- = \int \chi_{sys}^-(u)p(u)d(u)$ $\rightarrow$ Under **pessimistic** scenario
- $\Delta P = P^+ - P^-$ $\rightarrow$ Maximum deviation

We can now **quantify** the total level of uncertainty in the Probability of Systemic Default: ΔP

Case Study: Ring Market

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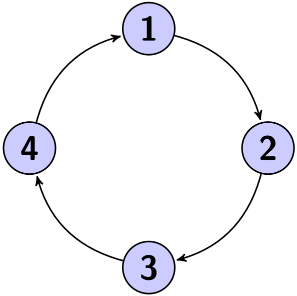

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Other Results

- **Comparative statics** between different structures: Ring vs Star
  - $\Delta_{ring} P < \Delta_{star} P$
  - Increase of cycles

- Effect of **correlation** on uncertainty: Non-monotonous role
  - Homogenous case: correlation increases uncertainty
  - Heterogenous case: correlation both increases and decreases uncertainty

- Express in terms of **expected losses**

$$E^{sys}_{loss} = \int \sum_i \omega_i(\varepsilon_i + \beta_i - \gamma_i - 1)\chi^*_i(u)p(u)du$$