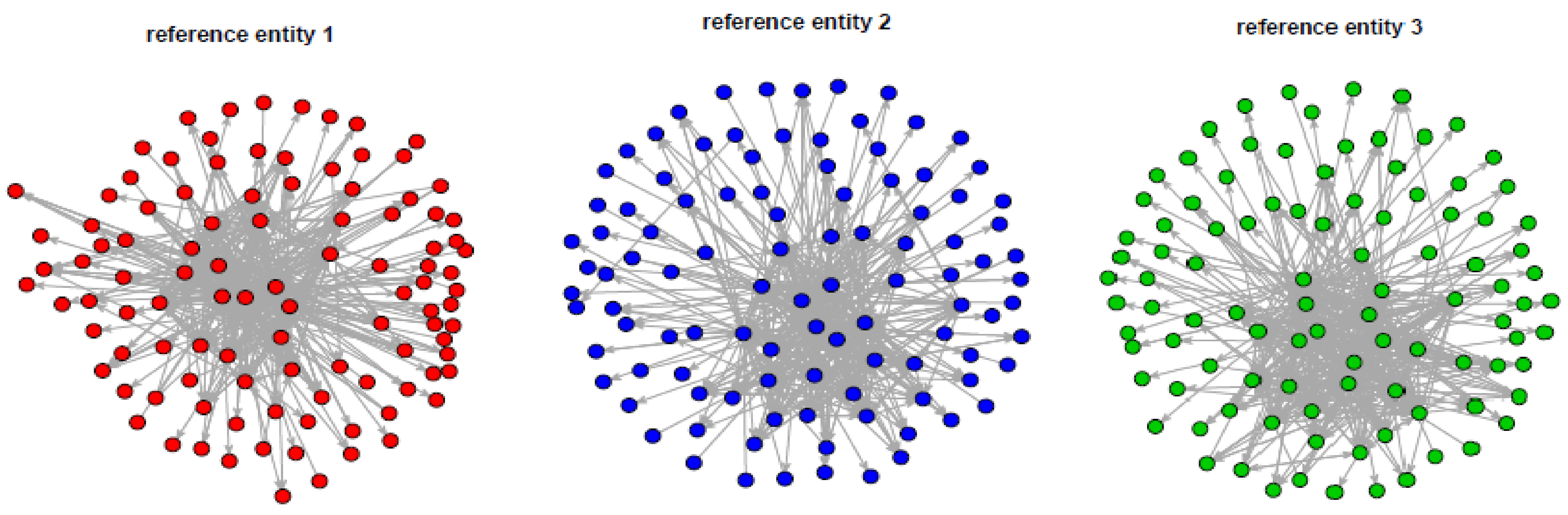
An Empirical Analysis of Network Reconstruction Methods using UK CDS Networks

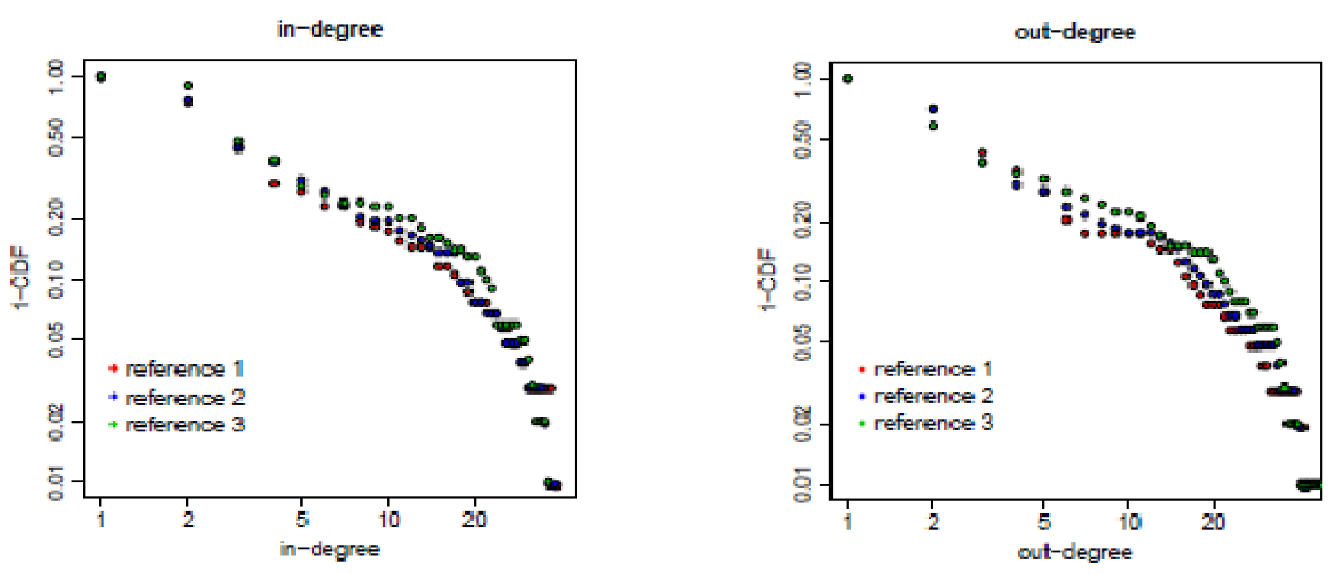
W. Abel and L. Silvestri
Bank of England

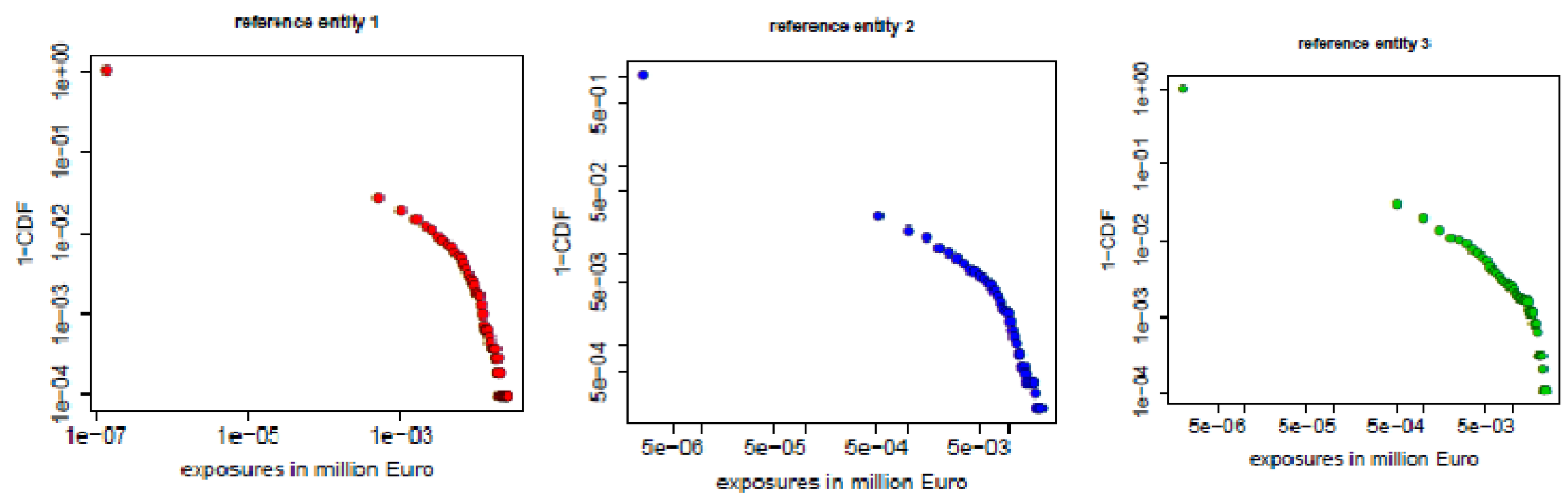
Aim of the work

- This work is the first to tackle the problem of reconstructing networks of bilateral exposures in OTC derivative markets when the partial data are provided by the new trade reporting rules.
- Trade reporting rules have been introduced by G20 leaders in the aftermath of the crisis and OTC market participants are required to report transaction level data to Trade Repositories (TRs), which are accessible to appropriate regulators.
- We propose a modification of the Cont and Moussa method in order to incorporate data newly available to regulators thanks to trade reporting rules.

Empirical Analysis of UK CDS networks







- UK CDS networks are heterogeneous and sparse.
- The more interconnected component is composed by the dealers; the less interconnected one by their clients.

Cont and Moussa Reconstruction Method

- This method relies on the assumption that UK CDS networks are well approximated by weighted directed scale-free networks.
- It consists in generating an ensemble of M weighted directed scale-free random networks $y^{(1)}, \ldots, y^{(M)}$ using the preferential attachment model of Bollobás et al. (2003), and assigning Pareto distributed weights to the edges.
- Since these networks do not achieve the in- and out-strengths constraints, they are weighted with probabilities $p_1, \ldots, p_M$ in such a way that the constraints posed by the available data are satisfied on the average across these probabilities.
- This method gives a distribution of random networks that satisfies on average the constraints posed by the available data.

$$\inf_{\boldsymbol{p} \in \mathbb{R}^k} H(\boldsymbol{p}) = \sum_{k=1}^{M} p_k \log M p_k$$

s.t.

$$\mathbb{E}^{\boldsymbol{p}}\left[\sum_{j=1}^{n} y_{ij}^{(k)}\right] = \sum_{k=1}^{M} p_k \left[\sum_{j=1}^{n} y_{ij}^{(k)}\right] = s_i^{out} \quad i = 1, \cdots, n$$

$$\mathbb{E}^{\boldsymbol{p}}\left[\sum_{j=1}^{n} y_{ji}^{(k)}\right] = \sum_{k=1}^{M} p_k \left[\sum_{j=1}^{n} y_{ji}^{(k)}\right] = s_i^{in} \quad i = 1, \cdots, n.$$

Cont and Moussa Reconstruction Method with Trade Reporting Rules

The new trade reporting rules add the knowledge of all bilateral exposures of m financial institutions.

$$\overbrace{\left(\sum_{j=1}^{n} x_{1j} \quad \cdots \sum_{j=1}^{n} x_{mj} \quad \cdots \quad \sum_{j=1}^{n} x_{nj}\right)}^{s^{in}}$$

$$\boldsymbol{x} = \begin{pmatrix} x_{11} & \cdots & \cdots & \cdots & \cdots & \cdots & x_{1n} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ x_{m1} & \cdots & \cdots & \cdots & \cdots & \cdots & x_{mn} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} & \cdots & \cdots & \cdots & \cdots \end{pmatrix} \left.\begin{pmatrix} \sum_{j=1}^{n} x_{j1} \\ \vdots \\ \sum_{j=1}^{n} x_{m1} \\ \vdots \\ \sum_{j=1}^{n} x_{jn} \end{pmatrix}\right\} s^{out}$$

$$\inf_{\boldsymbol{p} \in \mathbb{R}^k} H(\boldsymbol{p}) = \sum_{k=1}^{M} p_k \log M p_k$$

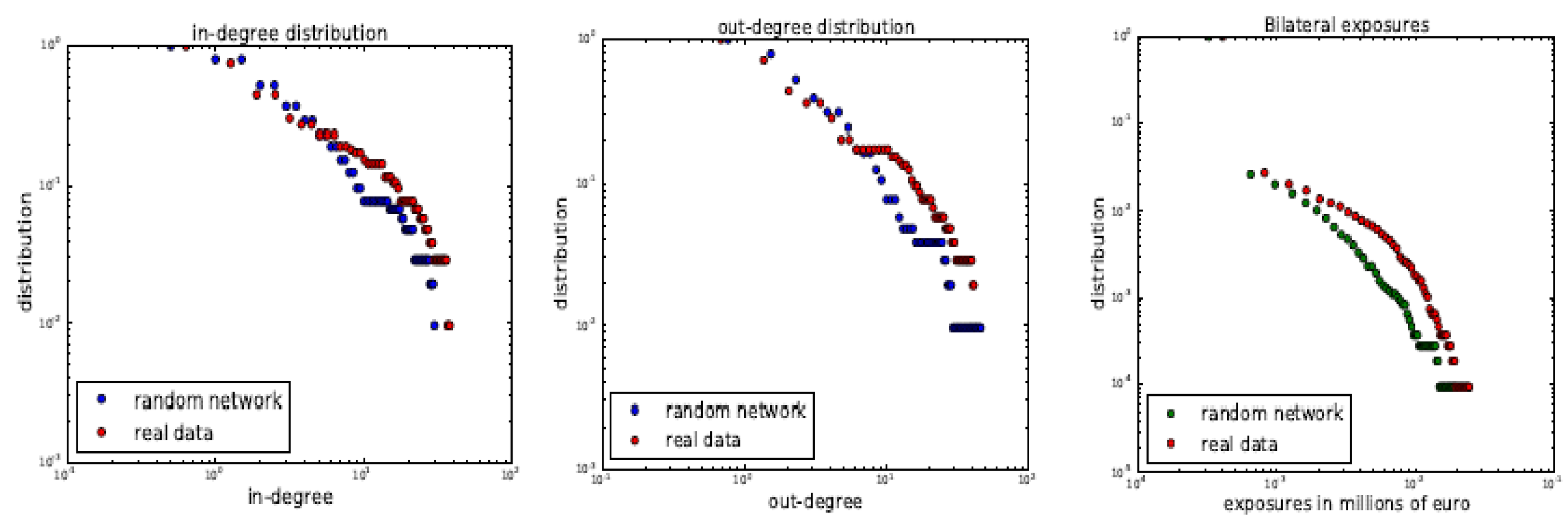
s.t.

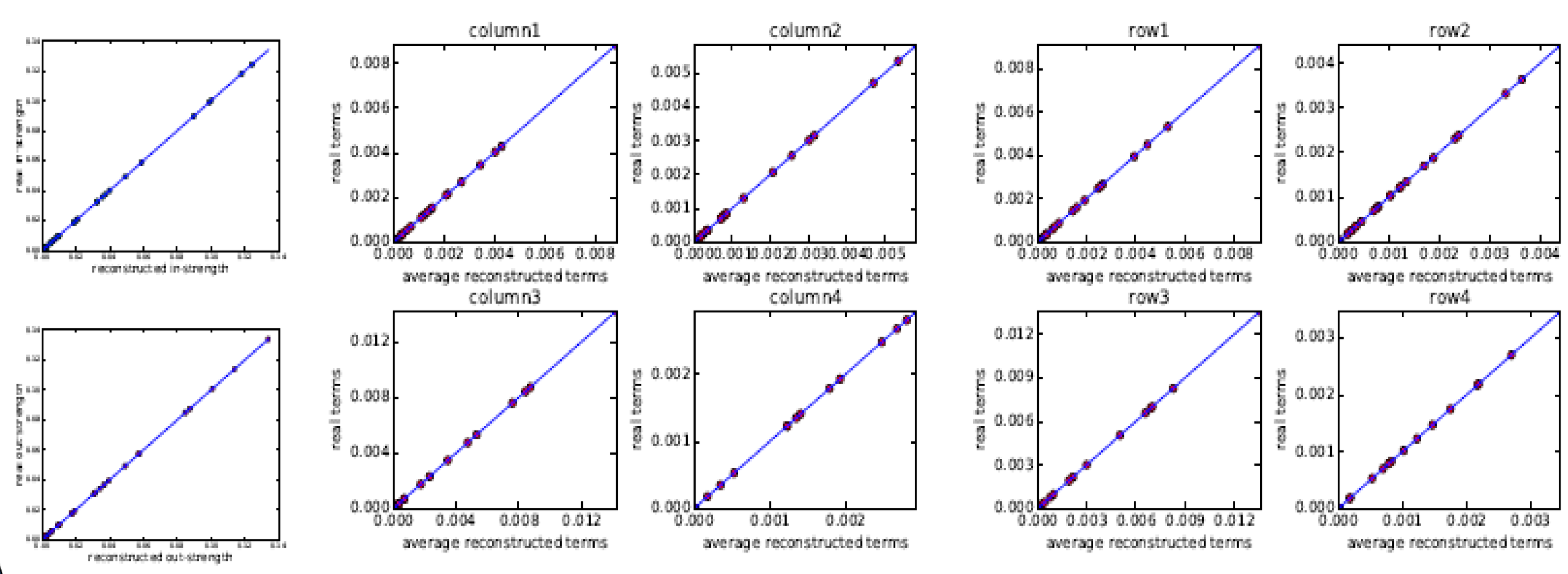
$$\mathbb{E}^{\boldsymbol{p}}\left[\sum_{j=1}^{n} y_{ij}^{(k)}\right] = \sum_{k=1}^{M} p_k \left[\sum_{j=1}^{n} y_{ij}^{(k)}\right] = s_i^{out} \quad i = 1, \cdots, n$$

$$\mathbb{E}^{\boldsymbol{p}}\left[\sum_{j=1}^{n} y_{ji}^{(k)}\right] = \sum_{k=1}^{M} p_k \left[\sum_{j=1}^{n} y_{ji}^{(k)}\right] = s_i^{in} \quad i = 1, \cdots, n$$

$$\mathbb{E}^{\boldsymbol{p}}\left[y_{ij}^{(k)}\right] = \sum_{k=1}^{M} p_k y_{ij}^{(k)} = x_{ij} \quad \text{for } i = 1, \ldots, m \;\; j = 1, \ldots n$$

$$\mathbb{E}^{\boldsymbol{p}}\left[y_{ji}^{(k)}\right] = \sum_{k=1}^{M} p_k y_{ji}^{(k)} = x_{ji} \quad \text{for } i = 1, \ldots, m \;\; j = 1, \ldots n$$





Conclusions

The networks so reconstructed can be used to evaluate any systemic risk indicator ϕ as the average across the network distribution

$$\phi = \mathbb{E}^{\boldsymbol{p}}[\phi] = \sum_{k=1}^{M} p_k \phi(k).$$

- More complexity can be added to explore other cases of incomplete information on bilateral exposure accessible to regulators.
- Policy implications: further work can help to determine the improvements made by trade reporting rules in the ability of regulators to reconstruct networks of bilateral exposures, or to better design data sharing agreements.

References

- A. Moussa. Contagion and systemic risk in financial networks. PhD Thesis, Columbia University, 2011.
- B. Bollobás, C. Borgs, J.Chayes and O. Riordan. Directed scale-free graphs. Proceedings of the fourteenth annual ACM-SIAM symposium on Discrete algorithms, pages 132-139, 2003.