Monitoring Indirect Contagion*

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Abstract

Large overlapping portfolios can become important amplifiers of stress across financial markets during times of crisis. How can the contagion risk posed by the distressed liquidation of such portfolios be monitored? More generally, how can one *quantify* the notion of interconnectedness that common asset holdings create? Our paper introduces two novel indicators, derived from the network of liquidity-weighted portfolio overlaps, to address these questions.

The Endogenous Risk Index (ERI) captures spillovers across portfolios in scenarios of deleveraging and has a natural micro-foundation that arises when accounting for institution-level losses occurring in a fire sale. The Indirect Contagion Index (ICI) allows to quantify the degree of "interconnectedness" for systemically important financial institutions' portfolios by accounting for the losses that a distressed liquidation would inflict on *other* portfolios.

Using data on 51 European banks from the European Banking Authority, we show that our indicators provide valuable information above and beyond the size of banks. These findings are robust to a wide range of modeling assumptions. Therefore our indicators may be useful to quantify the degree of "interconnectedness" in the Basel Committee on Banking Supervision's Indicator Measurement Approach, as well as contribute to the global monitoring of contagion channels.

Keywords: Financial stability, price-mediated contagion, macro prudential regulation, systemic risk measurement.

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1 Introduction

How can the notion of interconnectedness that common asset holdings create between institutions, and SIFIs in particular, be quantified? Which institutions are the most important propagators of stress in a deleveraging scenario? How can one monitor the risk of contagion in networks with overlapping portfolio? These are questions that we seek to address in this paper.

There is a rich theoretical literature addressing the analysis of overlapping portfolio networks: (Ibragimov et al., 2011) shows that diversification that may be optimal from an institution's individual point of view can create enormous systemic risks for the system. Their model illustrates that the optimal social outcome depends critically on the distributional properties of returns, and in general involves less risk-sharing than would be optimal from an individual perspective. A similar point is made by (Beale et al., 2011), who quantify in simple models of three to five assets how the regulator's preference for portfolio *diversity* may be at odds with the institutions' individual preferences for *diversification*. (Caccioli et al., 2014a) derive asymptotic properties of deleveraging cascades in bipartite asset-institution networks using Galton-Watson processes. Highly connected portfolio networks exhibit a "stable yet fragile" behaviour, i.e. contagion is generally unlikely to occur, but will be catastrophic if it does. (Wagner, 2011) shows that in the presence of liquidation risks, it is rational for investors to choose heterogeneous portfolios, i.e. they rationally choose to forgo diversification benefits, so as to reduce the risk of joint liquidations.

Detailed empirical analyses provide evidence of the effects predicted by theoretical models: (Khandani and Lo, 2011) trace the substantial losses that a number of funds suffered during the "Quant event of August 2007" back to the distressed liquidation of a similarly composed portfolio. (Cont and Wagalath, 2016) go one step further by reverse-engineering the portfolio that one would have needed to liquidate in order to generate the observed changes in returns, covariances and eventually losses for similar portfolios. The Quant event of 2007 is thus a textbook example of price-mediated contagion: institutions that may have no relation to each other at all suddenly become "indirectly" connected (Cont and Schaanning, 2016). This connection arises by virtue of their portfolios being composed to varying degrees of the same assets, which opens the channel for mark to market losses to materialize on all of them simultaneously when a large market player is forced to liquidate (a part of) its position. (Cont and Wagalath, 2013) extend the theoretical study by (Pedersen, 2009) to quantify the impact of investors running for the exit, and the implications for asset returns, in a continuous time setting. At an international level, (Boyer et al., 2006) and (Jotikasthira et al., 2012) provide strong empirical support for the spillover effects that asset sales and purchases by large institutional investors can have. (Ellul et al., 2011) study the forced divestment of downgraded bonds by insurance companies and the impact on prices. Consistent with the prediction made in (Shleifer and Vishny, 1992), the effects on prices is all the more important when the buyers (hedge funds in (Ellul et al., 2011)) are constrained.

A number of measures have been developed to capture these effects more precisely. Indeed, the growing availability of securities holdings data on various market participants has sparked the interest in analysing systemic risk stemming from overlapping portfolios. Using security-level holdings data from the German Microdatabase Securities Holdings Statistics, (Timmer, 2016) provides evidence that banks and investment funds act pro-cyclically to price changes (they sell when prices fall and buy when prices rise), while insurance companies and pension funds act counter-cyclically. (Guo et al., 2015) analyse the topology of common holdings in the US fund industry and find that greater overlap contributes to greater negative excess returns in the presence of liquidations. Similarly, (Braverman and Minca, 2016) use a similar network of portfolio overlaps to derive three measures of vulnerability which are shown to correlate significantly with negative returns of funds under stress. The measures of indirect contagion that we introduce in this paper are using similar building blocks as the measures of vulnerability in (Braverman and Minca, 2016). Moreover, by relating it to the deleveraging model introduced in (Cont and Schaanning, 2016) we provide a micro-foundation for our measure of indirect contagion.

(Getmansky et al., 2016) introduce “cosine-similarity” (the scalar product of two portfolio’s weights) as a measure of similarity between portfolios. Their study shows that insurance companies with a higher measure of commonality tend to engage in larger common sales, regardless of portfolio size. We will compare our measures of indirect contagion in detail to cosine similarity, and other overlap measures. (Blei and Bakhodir, 2014) use clustering tools from machine learning to infer an ”Asset Commonality” measure of risk (ACRISK). The measure quantifies the degree of commonality between large portfolios and captures the build-up of systemic risk several years prior to the crisis.

(Kritzman et al., 2011), who use the degree of variation explained by the first principal component of asset returns as a measure of systemic risk in markets, constitutes an intermediate step between the exposure-based measures described above and market-based measures of systemic risk. Among the most widely used market-based measures are “Δ CoVaR”(Adrian and Brunnermeier, 2016), “SRISK” (Brownlees and Engle, 2016), and the Marginal Expected Shortfall “(Acharya et al., 2017). These measures are constructed from a statistical analysis of the equity returns of listed companies, and are meant to aggregate many risk facets, as priced by the market, rather than focus specifically on overlapping portfolios. SRISK for instance measures the capital shortfall of a firm conditional on a severe market decline and depends on the firms size, leverage and risk. While not being based on actual exposures (and thus potential future losses), such measures aggregate market perceptions of risk into a number, and can function well as thermometers of risk. (Bisias et al., 2012) provides a comprehensive overview of different systemic risk measures.

By building on actual portfolio holdings, the contagion measures introduced in this paper function rather like a barometer of risk, quantifying “pressure” in the sense of the potential for future losses. The upside of our approach is that it allows to uncover risks that are not priced by the market because portfolio holdings are confidential and this risk cannot be priced in the market. The downside is of course that a sophisticated implementation of the methodology requires access to such confidential securities holding data. However, as part of the post-crisis reform package, such securities holdings data is currently being collected, which thus allows these analyses to be performed in the future.[1]

[1] See e.g. https://www.ecb.europa.eu/stats/financial_markets_and_interest_rates/securities_holdings/html/index.en.html

Main contributions.

1. **Endogenous Risk Index:** Given portfolio holdings data, we construct a network of *indirect contagion* which is based on the overlap of institutions' portfolios, adjusted for the liquidity of the underlying assets. We introduce the Endogenous Risk Index (ERI), the Perron-eigenvector of this liquidity-weighted overlap matrix of portfolios, and show that it has a natural micro-foundation that comes from correctly quantifying mark to market losses in deleveraging scenarios. The ERI is shown to be a good proxy of spillover losses over and above the size of portfolios. In particular, accounting for the liquidity of the underlying assets in the portfolio improves the prediction of fire sales losses markedly. The ERI can function as a proxy for monitoring contagion in periods between stress tests.

2. **Indirect Contagion Index:** The Indirect Contagion Index (ICI) is a modified version of the Endogenous Risk Index, which ignores self-inflicted liquidation losses and stresses losses caused to *other* portfolios. In the framework of the Basel Committee on Banking Supervision's GSIB methodology the ICI could be used as an additional indicator to *quantify* the degree of interconnectedness of systemically important institutions. Using data from the 2016 EBA stress test, we find that relating to the price-mediated contagion channel, Crédit Agricole, Deutsche Bank, BNP Paribas, Société Générale and Barclays are among the most systemic institutions.

The predictive power of our indicators is robust to a wide range of modeling assumptions such as the liquidation strategies which institutions use to delever portfolios, or the scenarios that trigger these.

Structure. The paper is organised as follows: Section 2.1 is devoted introduces our framework to monitor indirect contagion. Section 2.2 and 2.3 present the Endogenous Risk Index and the Indirect Contagion Index respectively. Sections 3.1 and 3.2 apply our methodology to a dataset of European Banks. Moreover, we perform a large sensitivity analysis in Section 3.3. and introduce an optimal bank response in Section 3.4. Section 3.5 compares the ERI and ICI to other measures of overlap, while Section 3.6 suggests an application of the indicators to monitor Global Systemically Important Banks (GSIBs) in the Basel Committe on Banking Supervisions' framework. Section 4 concludes.

2 Monitoring indirect contagion

2.1 Fire-sales losses and liquidity-weighted overlaps

We consider a financial network consisting of $i = 1..N$ institutions and $\mu = 1..M$ securities. The institutions are subject to regulatory, market- or self-imposed constraints, which may depend on their capital, leverage or liquidity levels. Let $\Pi^{i,\mu}$ denote the value of institution i's holdings in asset class μ in monetary units. The portfolio holdings of the entire system are given by the matrix $\Pi \in \mathbb{R}^{N\times M}$. This matrix corresponds to a bi-partite network of institutions and asset classes, as shown in Figure 1.

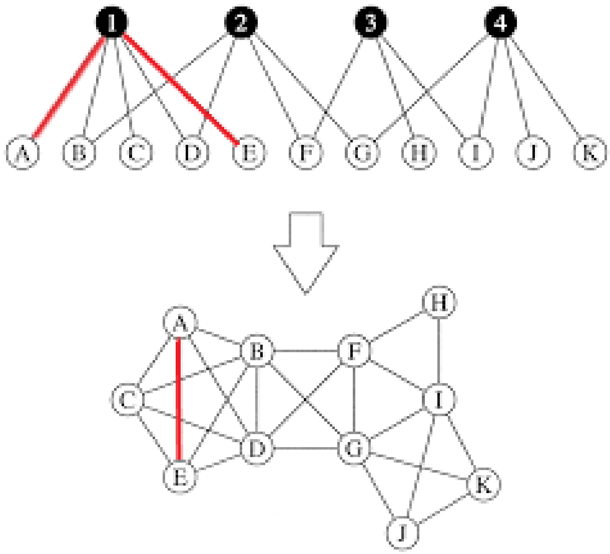

Figure 1: Bipartite network of institutions A-K and asset classes 1-4, which gives rise to a network of indirect contagion between institutions.

Such a network can give rise to *indirect* contagion: Even though institutions A through E may have no connection, institutions B to E may suffer losses if for some reason A is forced to liquidate its position in asset class 1, thereby depressing its price. Specifically, two types of losses can arise: (i) mark-to-market losses on remaining holdings, suffered by all parties that hold the asset undergoing the distressed liquidation and (ii) implementation costs which the liquidating institution suffers on the position it is trying to liquidate. Moreover, if, as a result of the forced deleveraging by A, institutions B or D need to liquidate a part of their portfolio, the price of asset class 2 may drop and cause losses for institutions F and G, who were previously unaffected. This shows how overlapping portfolios can become a driver of cross-asset class contagion, even when the asset classes are economically and geographically essentially independent.

In order to quantify this phenomenon properly, we need to specify: (i) *when* institutions react to a shock, (i) *how* they react when forced to, and (iii) *how* prices respond to forced liquidations. Using these three building blocks, we show that an endogenous risk index naturally arises when one quantifies the liquidation losses in such a network of constrained and overlapping portfolios.

The presentation follows the model discussion in Cont and Schaanning (2016), to which we refer the reader for further details:

1. At time $k = 1$, in response to a stress scenario, parametrised by percentage shocks to asset classes $\epsilon \in [0, 1]^M$, some institutions j deleverage their portfolio by selling a proportion $\Gamma^j_k(\epsilon) \in [0, 1]$ of marketable assets. In Section 3.4, we assume that banks do not sell assets proportionately, but choose Γ^j so as to minimise the liquidation losses they suffer. This leads to an aggregate amount $q^\mu_k = \sum_{j=1}^N \Pi^{j,\mu}_{k-1}\Gamma^j_k(\epsilon)$ of sales in asset class μ. In general, the deleveraging proportion, $\Gamma^j_k(\epsilon)$ can depend on the capital buffer, the liquidity reserves and other balance sheet components of institution j. In particular, when j is well-capitalised and has a prudent liquidity buffer, it will be able to withstand the shock without resorting to distressed liquidations, in which case $\Gamma^j(\epsilon) = 0$. We specify Γ precisely in the empirical

section and will drop the ϵ in $\Gamma(\epsilon)$ for notational convenience.

2. The market price of asset class μ is denoted by S_k^μ. The impact of asset sales results in a decline of the market price, moving it to

$$S_k^\mu = S_{k-1}^\mu \left(1 - \frac{q_k^\mu}{D_\mu}\right), \tag{1}$$

where the **market depth** $D_\mu(\tau) \propto \frac{ADV_\mu}{\sigma_\mu}\sqrt{\tau}$ is proportional to the ratio of the average daily volume (ADV) and the volatility of the asset class times the liquidation horizon, τ, which is assumed to be $\tau = 20$ days in our calibration. This corresponds to a linear price impact function $\frac{\Delta S}{S} = -\Psi_\mu(x) := \frac{x}{D_\mu}$, as used in ((Obizhaeva, 2012), (Kyle and Obizhaeva, 2016), (Amihud, 2002), (Cont and Wagalath, 2016)). For a more general discussion on non-linear price impact functions in the context of stress testing, we refer to (Cont and Schaanning, 2016).

3. For any institution i, the combined effect of its own deleveraging (if it occurs) and the impact of other forced sales changes the market value of its holdings in asset class μ to

$$\Pi_k^{i,\mu} = \underbrace{(1 - \Gamma_k^i)}_{\text{Remainder after deleveraging by } i} \overbrace{\Pi_{k-1}^{i,\mu}}^{\text{Previous value}} \underbrace{\left(1 - D_\mu^{-1} \sum_{j=1}^{N} \Gamma_k^j \Pi_{k-1}^{j,\mu}\right)}_{\text{Price impact on remaining holdings}}. \tag{2}$$

Importantly, this shows that in general the value of institution i's holdings under stress cannot be simply inferred from historically observed returns and covariances, but may depend on the stress scenario and the network of overlapping portfolios.

The price move generates two types of losses for portfolio i: First, it causes *mark to market losses* on the remaining part of the portfolio given by

$$\begin{aligned} M_k^i &= \sum_{\mu=1}^{M} \left((1 - \Gamma_k^i)\Pi_{k-1}^{i,\mu} - \Pi_k^{i,\mu}\right) = (1 - \Gamma_k^i) \sum_{\mu=1}^{M} \Pi_{k-1}^{i,\mu} D_\mu^{-1} \left(\sum_{j=1}^{N} \Gamma_k^j \Pi_{k-1}^{j,\mu}\right) \\ &= (1 - \Gamma_k^i) \sum_{j=1}^{N} \sum_{\mu=1}^{M} \frac{\Pi_{k-1}^{i,\mu} \Pi_{k-1}^{j,\mu}}{D_\mu} \Gamma_k^j. \end{aligned} \tag{3}$$

A second source of loss stems from the fact that assets are not liquidated at the current market price but at a discount. We assume for simplicity that the deleveraging institutions suffer the same price impact on the liquidated part of the portfolio as on their remaining part, yielding the *realised loss*

$$\begin{aligned} R_k^i &= \sum_{\mu=1}^{M} \left(\Gamma_k^i \Pi_{k-1}^{i,\mu} - \Gamma_k^i \Pi_{k-1}^{i,\mu} \left(1 - \frac{q_k^\mu}{D_\mu}\right)\right) \\ &= \Gamma_k^i \sum_{j=1}^{N} \sum_{\mu=1}^{M} \frac{\Pi_{k-1}^{i,\mu} \Pi_{k-1}^{j,\mu}}{D_\mu} \Gamma_k^j. \end{aligned} \tag{4}$$

Summing (4) and (3) yields the total loss of portfolio i at the k-th round of deleveraging:

$$L_k^i = \sum_{j=1}^{N} \underbrace{\sum_{\mu=1}^{M} \frac{\Pi_{k-1}^{i,\mu}\Pi_{k-1}^{j,\mu}}{D_\mu}}_{\Omega_{ij}(\Pi_{k-1})} \Gamma_k^j = \sum_{j=1}^{N} \Omega_{k-1}^{ij}\Gamma_k^j,$$

which shows that the magnitude of fire sales spillovers from institution i to institution j is proportional to the *liquidity-weighted overlap* Ω_{ij} between portfolios i and j (Cont and Wagalath, 2013):

$$\Omega_{ij}(\Pi_k) := \sum_{\mu=1}^{M} \frac{\Pi_k^{i,\mu}\Pi_k^{j,\mu}}{D_\mu}.$$

Let $D := \mathrm{diag}(D_1, \ldots, D_M)$, then the matrix of portfolio overlaps

$$\Omega(\Pi_k) = \Pi_k D^{-1} \Pi_k^\top, \tag{5}$$

can be viewed as a (liquidity-)weighted adjacency matrix of the network linking portfolios through their common holdings.

2.2 The Endogenous Risk Index.

From the derivation above, it follows that in the first round, the fire-sales losses for all banks are given by the vector

$$FLoss = \Omega\Gamma. \tag{6}$$

As Ω is symmetric positive semi-definite by construction, we know that there exists an orthonormal basis of eigenvectors with real eigenvalues for it. We further assume that Ω is an irreducible and non-negative matrix. This is equivalent to the network of overlapping portfolios being strongly connected (i.e. it is not a union of disjoint sub-networks) and that there are no pairs of portfolios such that $\sum_{\mu=1}^{M} \frac{\Pi^{i,\mu}\Pi^{j,\mu}}{D_\mu} < 0$ respectively. The European banking network which we use in our empirical analysis verifies these assumptions. Under these conditions, the Perron-Frobenius theorem ensures that the components of the first eigenvector are all positive.

If the first eigenvalue dominates, the network of liquidity-weighted portfolio overlaps can be approximated well by a one-factor model

$$\Omega \approx \lambda_1 u u^\top, \tag{7}$$

where u is the Perron eigenvector corresponding to the largest eigenvalue λ_1. Using this approximation, the fire-sales loss can be written as

$$FLoss^i = \lambda_1 u_i \sum_{j=1}^{N} u_j \Gamma_j(\epsilon).$$

Taking logarithms, we get

$$\begin{aligned}\log(FLoss^i) &= \log(\lambda_1 u_i \sum_{j=1}^{N} u_j \Gamma_j(\epsilon)) \\ &= \underbrace{1\times}_{\text{slope}} \log(u_i) + \underbrace{\log(\lambda_1) + \log(u^\top \Gamma(\epsilon))}_{\text{Intercept}}, \end{aligned} \tag{8}$$

which implies that the logarithm of the Perron eigenvector should be a good predictor of fire-sales losses. Moreover, we should expect a slope of approximately one when regressing log-fire-sales losses on it.

Equation (8) motivates the introduction of the following definition: The **Endogenous Risk Index (ERI)** of a financial institution is its component i in the Perron eigenvector u_i of the matrix Ω from (5) of liquidity-weighted portfolio overlaps:

$$ERI(i) = u_i. \tag{9}$$

The EIR provides a measure of centrality of the node i in the network whose links are weighted by the overlap matrix Ω. The ERI is constructed as follows:

1. Collect portfolio holdings $\Pi^{i,\mu}$ by asset class for each financial institution in the network, at the granularity level corresponding to bank stress tests.
2. Estimate a market depth parameter $D_\mu \propto \frac{ADV_\mu}{\sigma_\mu}$ for each asset class.
3. Check that $\Omega_{ij} \geq 0$ and that Ω is irreducible.
4. Compute the largest eigenvalue and the corresponding eigenvector (the "Perron eigenvector") $u = (u_i, i = 1...N)$ of the matrix of liquidity-weighted overlaps $\Omega(\Pi) = \Pi D^{-1}\Pi^\top$.
5. The Endogenous Risk Index is the Perron eigenvector, $ERI = u$.

2.3 The Indirect Contagion Index

Before turning to the empirical application, we propose a modification relative to the ERI that discounts the fire-sales losses that a distressed portfolio liquidation would inflict on itself, and only take into consideration the spillover-losses that are generated to *other* portfolios. We call this measure the **Indirect Contagion Index** (ICI), and compute it as follows:

1. Compute $\Omega = \Pi D^{-1}\Pi^\top$, as detailed for the ERI above.
2. Compute the largest eigenvalue and the corresponding Perron eigenvector v of $\Omega_0 := \Omega - \text{diag}(\Omega_{11}, \ldots, \Omega_{NN})$.[2]
3. The Indirect Contagion Index is the Perron eigenvector, $ICI = v$.

[2]Note that the zero diagonal of Ω_0 does not make the matrix reducible or violate the non-negativity constraint, and hence the Perron-Frobenius Theorem is still applicable.

We illustrate the difference between the ERI and the ICI, in a simple financial network of 7 banks and 2 asset classes given by:

$$\Pi = \begin{pmatrix} 1000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 100 & 1100 & 100 & 100 & 100 & 100 & 100 \end{pmatrix}^{\top}$$
$$D = (1000, 2000)^{\top}.$$

Banks 1 and 2 are thus large (and equal in size), while banks 3 - 7 are smaller. Asset class 1 is twice as *illiquid* as asset class 2. The left panel of Figure 2 shows the network, where the blue squares denote banks, the red circles denote asset classes and the size of the edge denotes the magnitude of the holding (not to scale). The bigger banks are denoted by larger squares; the more illiquid asset class is depicted by a larger circle. The right panel of Figure 2 shows the ICI (blue bars) and the ERI (black crosses) for this network. Bank 1 clearly dominates the ERI ranking. This is due to its main holdings being less liquid compared to Bank 2's holdings, and the amount of self-contagion that this could trigger. In contrast, Bank 2 has a large position in asset class 2, which all the medium-sized banks hold as well. Consequently, even though the same liquidation volume for Bank 2 in asset class 2 would cause a smaller loss than it would for Bank 1 in asset class 1, if Bank 2 delevers, the rest of the system also suffers significant fire-sales losses. The ICI discounts the self-inflicted fire-sales loss and only accounts for system externalities, which is why in the ICI ranking, Bank 2 dominates, and Bank 1 is equally ranked as the medium-sized banks.

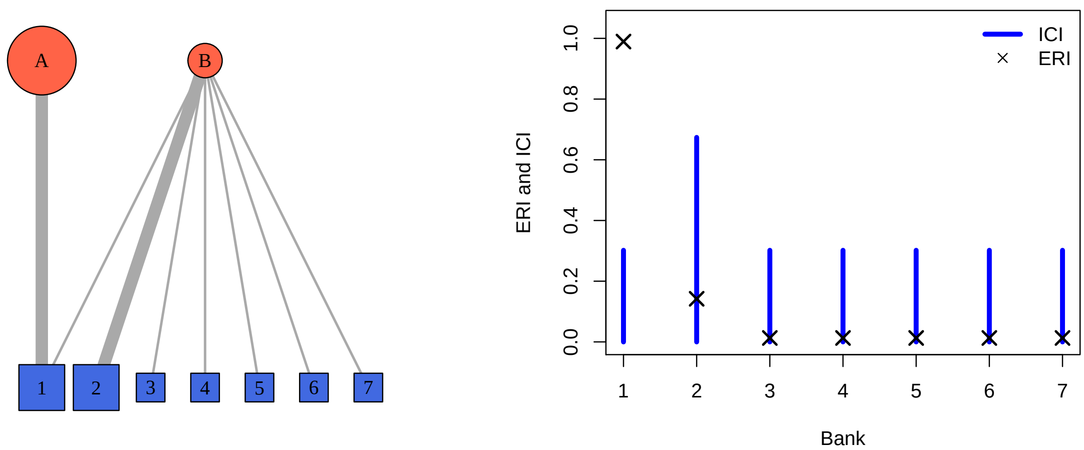

Figure 2: Illustrative example showing how the ICI discounts self-inflicted losses compared to the losses caused for other participants relative to the ERI.

We will further discuss the difference between the ERI and the ICI for European banks in the sections below.

3 Empirical Application

3.1 Data

We use data from the 2016 stress test exercise by the European Banking Authority (EBA) which provides information on notional exposures of 51 European banks across several hundred asset classes.[3] Holdings are disaggregated by asset type and geographical region. We subdivide marketable assets into corporate and sovereign bonds, which may be liquidated in a stress scenario. All other asset classes are classified as illiquid assets and assumed to be unavailable for short-term liquidations (non-securitised exposures). Ignoring asset classes to which the European banking system, as a whole, has an exposure below 1M EUR, leaves us with 93 asset classes, yielding a 51×93 matrix of liquid holdings Π. The most important regions for corporate exposures, covering over 75% of the total, are France (21.0%), U.S. (14.1%), Germany (11.7%), Italy (6.5%), Spain (4.6%), Netherlands (4.4%) and Belgium (3.2%). The most important regions for sovereign exposures, covering more than 75% of the total, are Germany (13.8 %), France (13.3%), U.S. (12.8%), Italy (9.2%), U.K. (8.4%), Spain (6.3%), Netherlands (4.6%), Belgium (4.2 %) and Japan (3.4%). Through a similar procedure, we obtain a 51×98 matrix of illiquid holdings, which we denote Θ. This corresponds to 97 asset classes for commercial and residential mortgage exposures respectively in the various regions and a 98th entry consisting of all remaining illiquid asset holdings (intangible goods, defaulted exposures etc).

Collectively, the 51 banks hold assets totalling 26.3 trillion EUR, of which 54.6 % (14.3 tn EUR) are in loans and advances, 31.1 % (8.2 tn EUR) are in marketable assets, and 14.3 % (3.8 tn EUR) are in “residual” assets classes that are not of relevance for our model.

The market depth parameter D_μ defined in (1) is computed following the methodology in (Obizhaeva, 2012) and (Cont and Schaanning, 2016). The estimation procedure is explained in more detail in Section 3.3, where we perform a sensitivity analysis on this parameter.

Empirical analysis. As the portfolio holdings from the 2016 EBA data fulfil all our assumptions, we compute Ω as well as the ERI and ICI as detailed above. The left-hand panel of Figure 3 shows that the distribution of liquidity-weighted overlaps Ω_{ij} displays considerable heterogeneity. The right panel shows the first twenty ranked eigenvalues of Ω. The first eigenvalue accounts for about 65% of the total variation and clearly dominates the remaining ones, we thus expect the ERI to be a good predictor of fire-sales losses in the sequel.

Figure 4 shows the Endogenous Risk Index for the European Banking network in 2016, where we have labelled the banks with the highest ERI: Crédit Agricole, BNP Paribas, Deutsche Bank, Société Générale and Barclays. All banks that are identified are either Global Systemically Important Banks (GSIBs) or Domestic Systemically Important Banks (DSIB). It is important to note that the banks are ranked by the amount of contagion they could trigger should they engage in distressed liquidations. This is *not* the same as ranking them according to their size – an issue we illustrate in detail in the next section.

Finally, Figure 5 depicts the indirect contagion network between European banks as implied by the EBA data. For visual purposes, we have placed the banks identified as the most systemic

[3] Cont and Schaanning (2016) uses the same data and follow their presentation here.

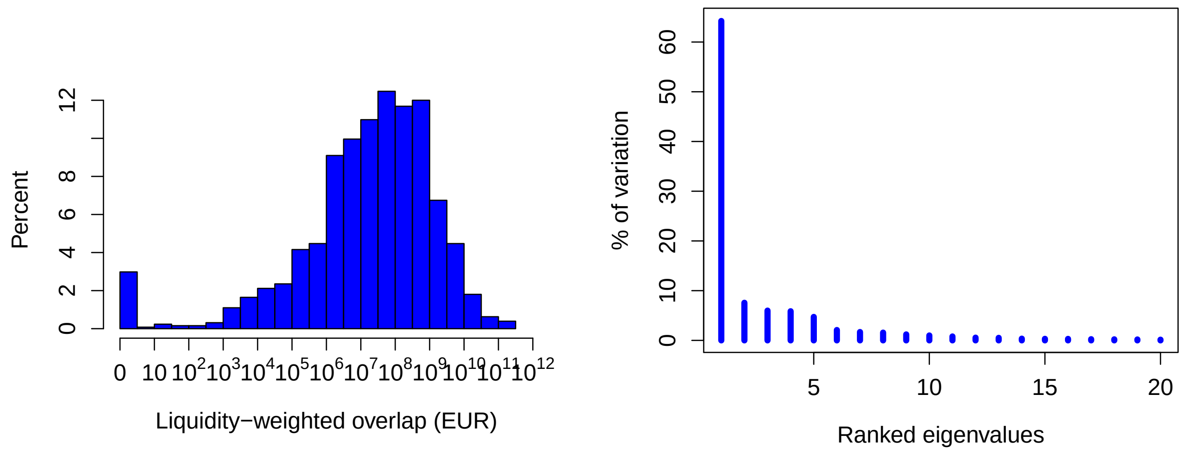

Figure 3: Distribution of liquidity-weighted overlaps (left) and the ranked eigenvalues of the indirect contagion network.

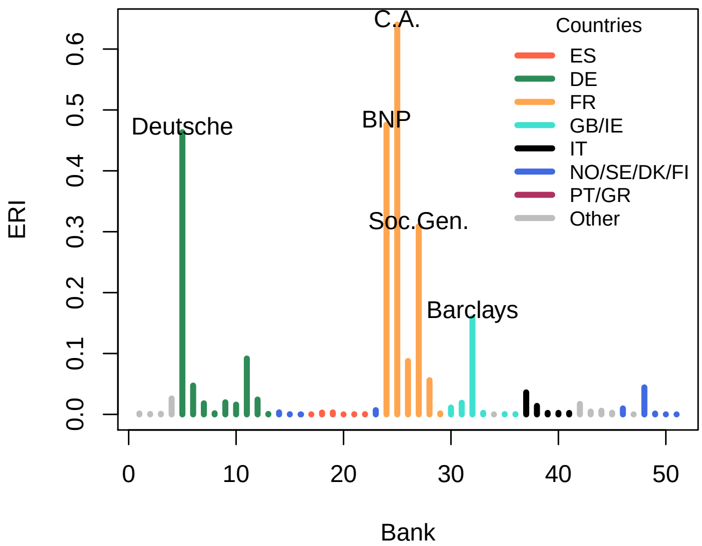

Figure 4: The Endogenous Risk Index for the European Banking system. (Data: EBA. Calculations: Authors).

by the ERI in the centre, and coloured edges orange that connect these banks with each other as well as with other banks.

3.2 Quantifying fire-sales losses with the Endogenous Risk Index.

Using the official loss rates of the EBA stress test as starting point, we estimate deleveraging that may occur as a result and the spillover losses that this may cause. Figure 6 shows the EBA scenario losses for the individual banks in percent of their capital. The 10 most severely hit banks are Banca Monte dei Paschi di Siena (MdPdS), Allied Irish Banks (AIB), Commerzbank (Com), The Royal Bank of Scotland (RBS), Bayerische Landesbank (BL), Raiffeisen-Landesbanken-Holding (RLH), Banco Popular Español (BPE), The Governor and Company of the Bank of Ireland (GCBI), Cooperatieve Centrale Raiffeisen-Boerenleenbank (CCRB)and Landesbank Baden-Württemberg (LBBW). As the EBA dataset does not provide information

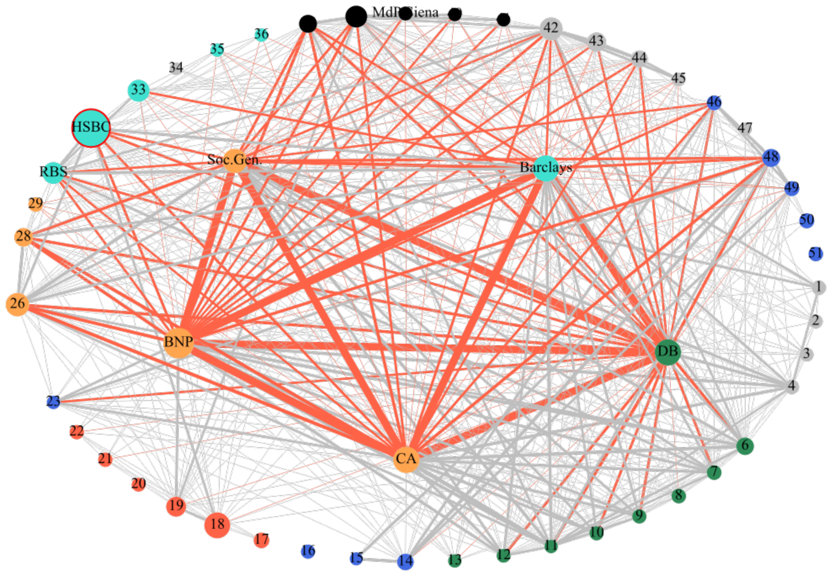

Figure 5: The EU indirect contagion network.

on the liquidity, or the risk weights of individual assets on the balance sheet, we focus on a leverage constraint only. Using supervisory data from the Bank of England, Coen et al. (2017) show how the model can be adapted to include risk-weighted capital and liquidity constraints. The initial leverage of bank i is $\lambda^i = (\sum_{\mu=1}^{M} \Pi^{i,\mu} + \sum_{\kappa=1}^{K} \Theta^{i,\kappa})/C^i$, where C^i is the capital of

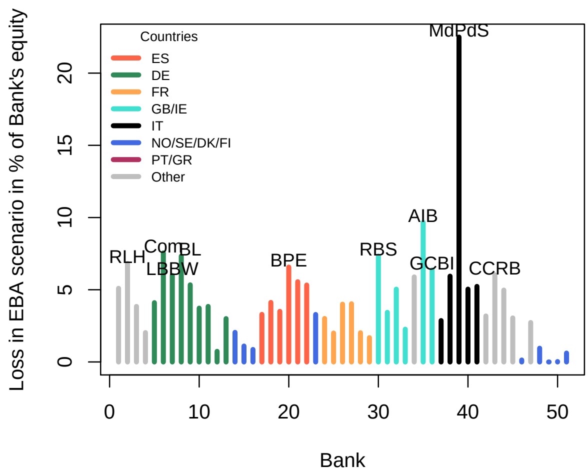

Figure 6: The losses in percent of bank equity that are estimated in the 2016 EBA adverse scenario.

bank i. After a loss l_i the leverage increases to

$$\lambda^i(\Pi, \Theta, C, l) = \frac{\sum_{\mu=1}^{M} \Pi^{i,\mu} + \sum_{\kappa=1}^{K} \Theta^{i,\kappa} - l_i}{(C^i - l_i)_+}. \tag{10}$$

If as a result of the shock the leverage λ^i exceeds the regulatory constraint $\lambda_{\max} = 33$, then the bank will liquidate a portion, $\Gamma^i \in [0, 1]$, of its marketable assets in order to become compliant again with the leverage constraint. The bank thus solves

$$\frac{(1-\Gamma^i)\sum_{\mu=1}^{M}\Pi^{i,\mu}+\sum_{\kappa=1}^{K}\Theta^{i,\kappa}-l_i}{(C^i-l_i)_+}=\lambda_b,$$

for Γ^i, where $\lambda_b \leq \lambda_{\max}$ is a buffer leverage. This yields

$$\Gamma^i(\Pi, C, \Theta, l) = \left(\frac{\sum_{\mu=1}^{M}\Pi^{i,\mu}+\sum_{\kappa=1}^{K}\Theta^{i,\kappa}-\lambda_b(C^i-l_i)}{\sum_{\mu=1}^{M}\Pi^{i,\mu}}\wedge 1\right)\mathbf{1}_{\lambda^{\mathbf{i}}(\mathbf{\Pi},\mathbf{\Theta},\mathbf{C},\mathbf{l})>\lambda_{\max}}. \tag{11}$$

In Section 3.4, banks will not simply sell assets proportionally but choose assets so as to minimise the liquidation losses they suffer. On the one hand if the fire-sales losses from the distressed liquidations of the first round cause other (or the same banks) to breach their leverage constraint (again), then one or more further rounds of deleveraging occur. On the other hand if the initial loss is not too severe and/or banks well capitalised, they may absorb the losses without any distressed sales occurring at all. Such a threshold-type reaction is typical of the asymmetric reaction of market participants to downside risk(Ang et al., 2006), sudden and non-linear event-driven deleveraging (Khandani and Lo, 2011), (Cont and Wagalath, 2016), trading-desk VaR constraints (Danielsson et al., 2012), and characteristic of systems of complex systems (Granovetter, 1978), (Duffie, 2010).

We follow the estimation procedure in (Obizhaeva, 2012) and (Cont and Schaanning, 2016), and estimate the market depth as $D_\mu(\tau) = c\frac{ADV_\mu}{\sigma_\mu}\sqrt{\tau}$, where $c = 0.3$ (Ellul et al., 2011) and $\tau = 20$ days. For the median asset, this yields an estimated impact of 440 basis points per 10 bn EUR liquidated. A sensitivity analysis on the market depth is performed in the next section. To compute the price impact during the fire sales cascade, $\frac{\Delta S_k^\mu}{S_k^\mu} = -\Psi_\mu(q_k^\mu)$, we use the non-linear two parameter market impact function

$$\Psi_\mu(q_k, S_k) = \left(1-\frac{B_\mu}{S_k^\mu}\right)\left(1-\exp(\frac{-q_k}{\delta_\mu})\right), \tag{12}$$

where $\delta_\mu = D_\mu(1 - B_\mu/S_0^\mu)$ ensures that the impact of small-volume sales match the observed impacts that are captured well by a linear model, and where B_μ is a floor below which the price does not fall. The floor accounts for the arrival of large institutional value-investors when prices drop far below fundamentals. This two-parameter impact model for stress testing was first used in (Cont and Schaanning, 2016) to which we refer for further details.

To be clear, we thus simulate the deleveraging cascade with a more realistic non-linear impact model, and use the linear approximation to compute the Endogenous Risk Index as detailed above. Table 1 shows the results from the regression specified in (8) for the estimated market depth (which results in a median impact of 440 bps per 10bn of assets liquidated). The results are encouraging: The coefficient of the slope is indeed close to 1, lying between 0.73 and 0.80 for the first three rounds. The R^2 ranges between 0.68 and 0.79 for the first four rounds. Even when considering the total fire-sales loss across all rounds, the regression still achieves an R^2 of 0.73 and has a slope of 0.62. All estimates are highly significant, with $p < 10^{-4}$. The first empirical results corroborate the theoretical suggestion that the ERI may be well suited to predict fire-sales losses. Additionally, Figure 13 in the Appendix shows the data and regression lines.

Table 1: Regression of bank-level fire-sales losses on the Endogenous Risk Index for all banks, as specified in (8).

	Round 1	Round 2	Round 3	Round 4	All rounds
Slope	0.730***	0.795***	0.752***	0.516***	0.623 ***
	(0.072)	(0.060)	(0.068)	(0.112)	(0.055)
Intercept	10.5***	10.9***	10.7***	9.76***	11.1***
	(0.190)	(0.151)	(0.164)	(0.326)	(0.143)
n	51	49	41	30	51
R^2	0.68	0.79	0.76	0.43	0.73

Is bank size sufficient to measure contagion? Next, we address the question whether the ERI captures information in that is not already included when considering the size of banks. The vector of normalised bank portfolio sizes is

$$Size = \frac{(\Pi^1, \ldots, \Pi^N)}{||(\Pi^1, \ldots, \Pi^N)||_2}, \tag{13}$$

where $\Pi^i := \sum_{\mu=1}^{M} \Pi^{i,\mu}$. Table 2 shows the R^2 and root-mean squared error (RMSE) when regressing log fire-sales losses on either the log size, log ERI and on both variables.[4] In this configuration, the ERI and size vector have essentially the same predictive power (R^2 of 0.77 and 0.79 respectively). More importantly, when regressing log fire-sales losses on both variables, they do both turn out highly significant ($p < 10^{-4}$), and the R^2 increases to 0.86, while the RMSE decreases from 0.33 to 0.27. Overall, the ERI thus clearly captures information over and above and beyond the mere size of bank portfolios. Next, we show that this result is robust against for a wide range of different model assumptions and all relevant parameter values.

Table 2: The ERI captures information above and beyond the size vector.

	Model 1	Model 2	Model 3
Size	0.98 ($p = 0.0000$)	–	0.58 ($p = 0.0000$)
ERI	–	0.54 ($p = 0.0000$)	0.27 ($p = 0.0000$)
R^2	0.79	0.77	0.86
RMSE	0.32	0.33	0.27
n	51	51	51

3.3 Sensitivity analysis: Scenario composition, market depth and shock intensity.

In order to exclude the possibility that the encouraging results for the ERI are due to a specific choice of the stress scenario or other model parameters such as the market depth and the shock intensity, we perform sensitivity analyses with respect to the each of these three factors.

[4]When size in (13) is defined based on total assets rather than securities holdings, the ERI has a higher R^2 and lower RMSE than Size.

Stress scenarios. To assess the sensitivity of our results with respect to the choice of stress scenario, we consider as in (Cont and Schaanning, 2016) three further scenarios:

1. **2016 EBA stress scenario:** We take the bank-level losses directly from the loss estimates of the 2016 EBA stress test.

2. **"Brexit" scenario:** Losses materialise on British and Irish residential exposures, as well as on the commercial exposures of the UK itself and its main European trading partners: Ireland, Germany, Norway and Italy;

3. **Southern European scenario:** Losses materialise on commercial and residential exposures in Italy, Spain and Portugal;

4. **Eastern European scenario:** Losses materialise on commercial exposures in Eastern Europe (Regions: Bulgaria, Czech Republic, Estonia, Hungary, Lithuania, Latvia, Poland, Romania and Slovakia);

The composition of the scenarios is arbitrary, with the intention to have different subsets of banks shocked in each scenario.

Stress intensity. In order to assess the sensitivity of the results with respect to the severity of the shock, we vary the shock intensity for all asset classes from 0 to 20% in each scenario.[5]

Market depth. In general, it is notoriously difficult to estimate the price-impact of a hypothetical liquidation under hypothetical market conditions. In particular, the task is not to establish a relative ranking of assets' liquidities, but to *quantify by how much* prices may realistically drop as a function of the liquidated amount. Unfortunately, this question has not yet received a significant amount of attention in the literature. We circumvent this difficulty by assessing the sensitivity of our results by using upper and lower bounds for the market impact, D_μ, similar to the uncertain volatility approach taken in (Avellaneda et al., 1995). More specifically, we scale the median impact from its estimated impact of 440 bps per 10 bn liquidated up to 2000 bps (i.e. less liquid markets than anticipated) as well as down to 50 bps (i.e. more liquid markets than anticipated). The upper half of Table 3 shows the estimated market depths for a number of selected corporate and sovereign asset classes for a 1bn and a 10 bn liquidation volume under the original estimate, as well as for the minimum and maximum impacts. The lower half of the table shows the median, average and weighted average (weighted by holdings) of the market depths.

Results. We run the regression (8) for all combinations of scenarios, market impacts and shock intensities. Each panel in Figure 7 shows the R^2 of the regression in each of the four scenarios as a function of the market depth and the shock intensity. The graphs show that across all combinations of scenarios, shock intensities and market depths, the regression is robust. For combinations of market depths and shock intensities where large-scale contagion occurs, the R^2

[5] The EBA scenario is calibrated such that a 10% shock corresponds exactly to the losses predicted in the official EBA stress test.

Table 3: Impact in basis points in selected asset classes as for 1 bn and 10 bn in sales.

Asset class	Impact in bps for **1 bn** sale Estimate	Minimum	Maximum	Impact in bps for **10 bn** sale Estimate	Minimum	Maximum
US gov	0.04	0	0.23	0.41	0.04	2.27
UK gov	0.52	0.06	2.87	5.18	0.56	28.63
DE gov	0.93	0.1	5.18	9.34	1.02	51.53
FR gov	1.73	0.19	9.58	17.27	1.89	94.97
IT gov	3.18	0.35	17.62	31.75	3.47	173.43
US corp	0.21	0.02	1.19	2.15	0.23	11.88
UK corp	2.68	0.29	14.84	26.74	2.92	146.4
DE corp	4.85	0.53	26.83	48.31	5.29	261.94
FR corp	9	0.98	49.68	89.28	9.81	475.19
IT corp	16.5	1.8	90.78	162.59	17.99	837.07
Median	46	5	249	**440**	**50**	**2000**
Mean	499	136	1086	1410	518	2591
Weighted av.	60	9	174	247	63	700
Median sov	26	3	142	253	28	1251
Mean sov	153	19	586	848	165	1855
Wghtd av. sov	11	1	53	90	11	358
Median corp	152	17	787	1327	166	4062
Mean corp	796	238	1516	1892	821	3224
Wghtd av. corp	100	16	274	378	107	984

is particularly predictive, with values above 0.75. However, for combinations of shock intensities and market depths that lead to localised fire sales(e.g. scenario 4), the R^2 is lower at around 0.4 - 0.5. Figure 8 illustrates that the slopes of the regressions range between 0.5 and 0.85 as a function of the market impact and shock intensity across the four scenarios.

Figure 15 of the appendix plots the fire-sales losses in each of the four scenarios as a function of the shock intensity and market depth, which illustrates that the cliffs in Figures 7 and 8 correspond to the boundaries between moderate versus considerable fire-sales losses.

As a rule of thumb, when contagion spreads far throughout the system, the predictive value of the ERI – as measured by the R^2 of the regression – increases substantially. For instance in Scenario 1, when the median impact is close to 2000 bps, the R^2 ranges between 0.7 and 0.9, while the slope lies between 0.76 and 0.96. Table 4 shows the results of these regressions in this high-impact region.

Table 4: Regression of bank-level fire-sales losses on the Endogenous Risk Index for all banks, as specified in (8) for the high-impact region.

	Round 1	Round 2	Round 3	Round 4	All rounds
Slope	0.758***	0.898***	0.957***	0.954***	0.774 ***
	(0.072)	(0.050)	(0.049)	(0.062)	(0.051)
Intercept	9.96***	10.2***	10.1***	9.81***	10.6***
	(0.188)	(0.134)	(0.118)	(0.148)	(0.135)
n	51	49	46	46	51
R^2	0.70	0.87	0.90	0.84	0.82

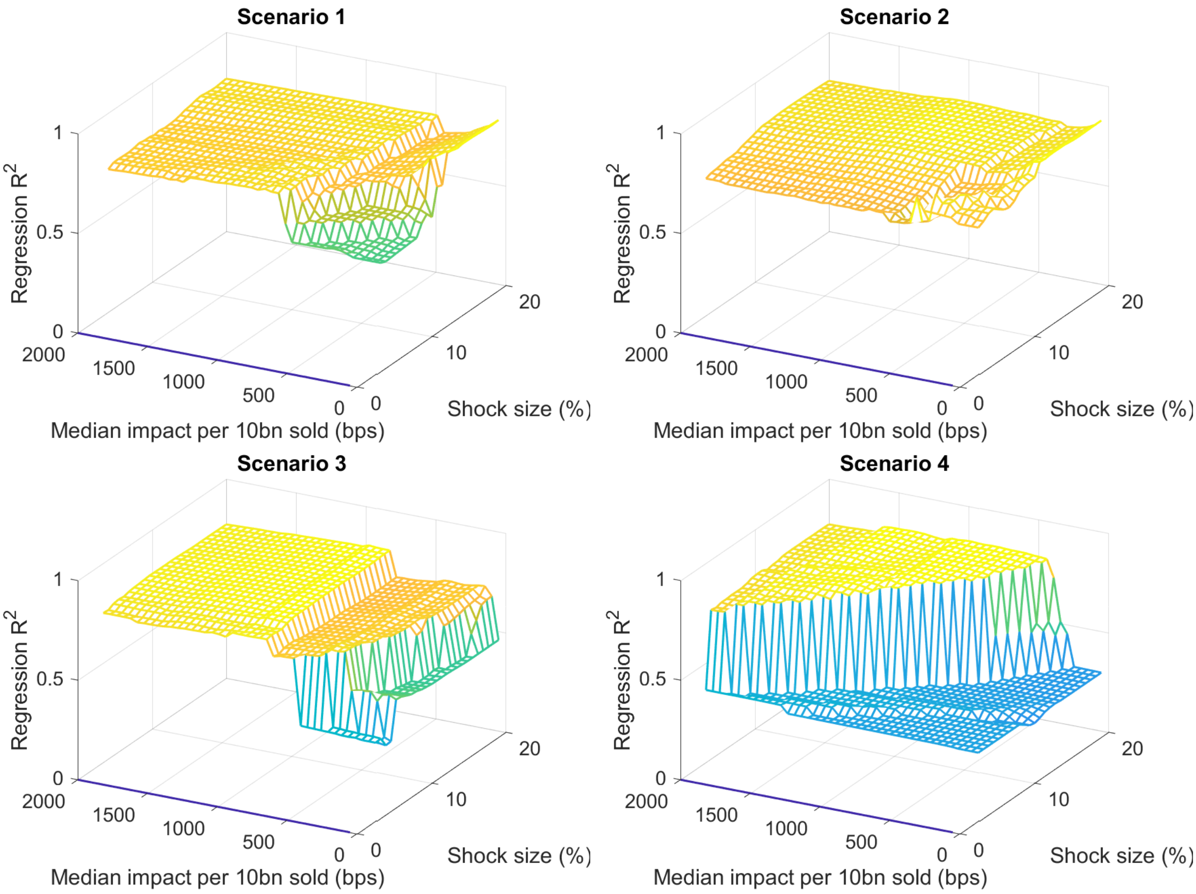

Figure 7: R^2 of the regression of fire-sales losses on the ERI as a function of the shock size and market depth in each scenario.

3.4 Loss-minimising portfolio deleveraging

Section 3.3 showed that our findings are robust to stress scenario composition, market depth and shock intensity. In this section, we extend the robustness check to the behaviour of banks. Previously, we assumed that deleveraging occurs proportionally across all assets. In the sequel, we assume, similar to the approach taken in (Coen et al., 2017) or (Baes et al., 2018), that a bank i needing to liquidate a part of its portfolio will do so by trying to minimise the fire-sales losses that it incurs:

$$\min_{\Gamma^i \in [0,1]^M} \sum_{\mu=1}^{M} \frac{\Pi^{i,\mu}\Pi^{i,\mu}}{\delta_\mu} \Gamma^{i,\mu} \tag{14}$$

subject to the bank complying with its leverage constraint

$$\frac{\sum_{\mu=1}^{M}(1-\Gamma^{i,\mu})\Pi^{i,\mu} + \sum_{k=1}^{K} \Theta^{i,k} - l_i - \sum_{\mu=1}^{M} \frac{\Pi^{i,\mu}\Pi^{i,\mu}}{\delta_\mu}\Gamma^{i,\mu}}{C^i - l_i - \sum_{\mu=1}^{M} \frac{\Pi^{i,\mu}\Pi^{i,\mu}}{\delta_\mu}\Gamma^{i,\mu}} \leq \lambda_{\max}. \tag{15}$$

As before, due to the absence of information on liquidity and risk weights in the public EBA data, we limit the exercise to the leverage constraint. (Baes et al., 2018)[Prop.1] show that in the case of a leverage constraint only with linear market impact, the solution to this problem is given by the following strategy.

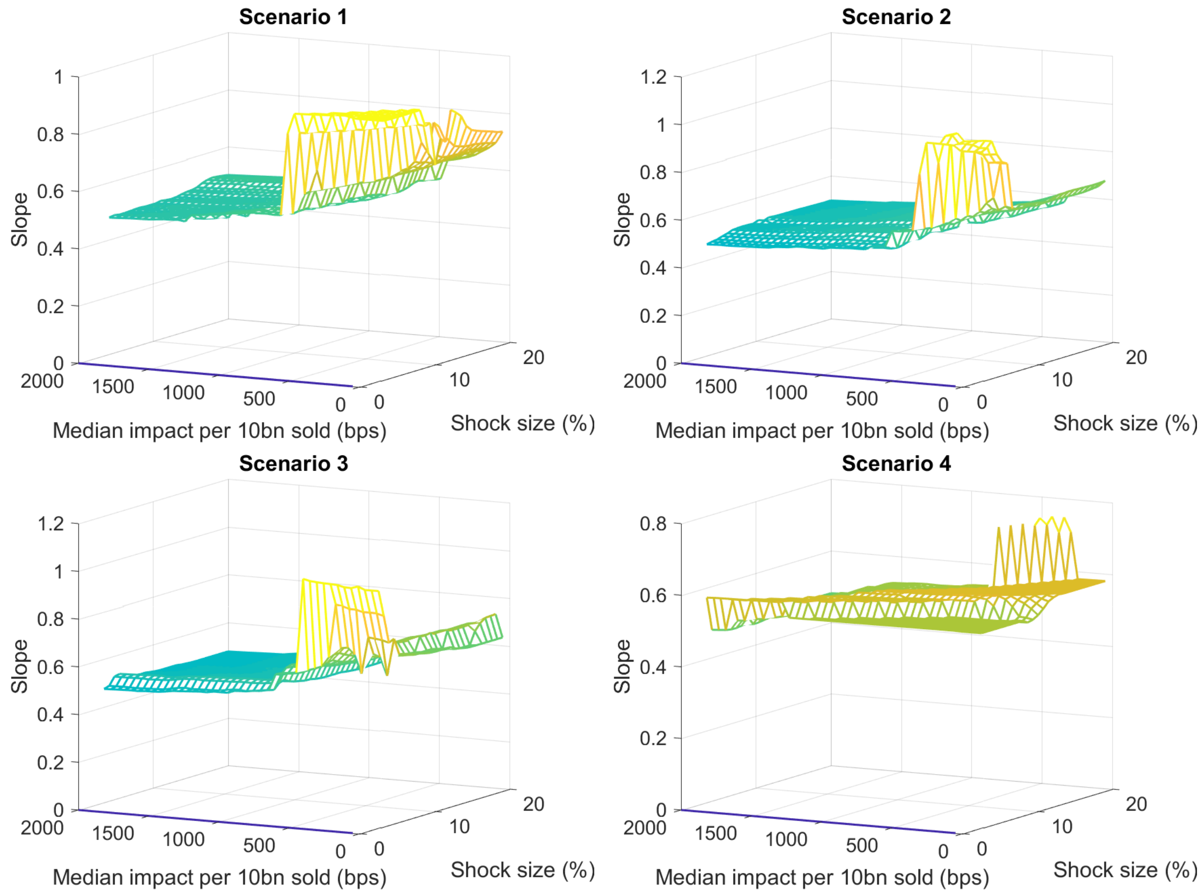

Figure 8: Slopes of the regression of the fire-sales losses on the ERI as a function of the shock size and market depth in each scenario.

Proposition 1. *Under a leverage constraint only and a linear market impact function, the individually optimal solution for an institution to delever its portfolio, as specified by the optimisation problem* (14) *and* (15)*, is to do so by selling its assets sequentially in decreasing order given by the ratios* $\frac{\delta_{\mu_1}}{\Pi^{i\mu_1}} > \ldots > \frac{\delta_{\mu_M}}{\Pi^{i\mu_M}}$ *until the constraint is fulfilled, or the bank runs out of marketable assets to delever (and defaults).*

Next, we analyse the predictive value of the ERI for fire-sales losses given that banks engage in such optimal sequential deleveraging. Table 5 reports the results for the estimated market depths (median impat of 440 bps per 10 bn). The slope lies between 0.60 and 0.66 in the first three rounds, and the R^2 lies between 0.42 and 0.60. In comparison to Table 1, the predictive power has become somewhat weaker but coefficients are still highly significant and the slope remains in close to 1. We performed the same sensitivity analyses as in the previous section on the scenarios, market depths and shock intensities when banks liquidate assets optimally. We find that across all scenarios, market depths and shock intensities the regression slope varies between 0.6 and 0.85, while the R^2 varies between 0.3 and 0.6. As results are similar to the case of proportional deleveraging, we relegate the corresponding figures to the Appendix (Figures 14, 16, 17, 18, and 19). Overall, the main contrast between the optimised versus the proportional deleveraging assumption is thus the magnitude of fire-sales losses, which is smaller when banks liquidate their portfolio in this way.[6]

Table 5: Regression of bank-level fire-sales losses on the Endogenous Risk Index for all banks, as specified in (8) for optimised bank responses.

	Round 1	Round 2	Round 3	Round 4	All rounds
Slope	0.614***	0.658***	0.600***	0.554***	0.613 ***
	(0.072)	(0.105)	(0.107)	(0.111)	(0.073)
Intercept	10.2***	8.75***	8.23***	7.76***	10.2***
	(0.190)	(0.281)	(0.288)	(0.301)	(0.191)
n	51	46	46	46	51
R^2	0.60	0.47	0.42	0.36	0.59

3.5 Comparison with other measures of overlap.

We have established that the *ERI* captures information above and beyond the size of bank portfolios. In this final section, we compare two further measures of portfolio similarity and overlap:

1. **Cosine Similarity.** We follow (Getmansky et al., 2016), and define the weights of a portfolio as the relative proportion of wealth invested in the various assets

$$w_i := \frac{1}{\sum_{\mu=1}^{M} \Pi^{i,\mu}} (\Pi^{i,1}, \ldots, \Pi^{i,M})^\top. \tag{16}$$

(Getmansky et al., 2016) proceed to define a measure of similarity, called "cosine similarity" between two portfolios via

$$\Omega^{ij}_{C.S.} = \frac{< w_i, w_j >}{||w_i||_2 ||w_j||_2} \in [-1, 1] \,. \tag{17}$$

The cosine-similarity measure is computed as the first eigenvector of $\Omega_{C.S.}$.

2. **Nominal Overlap.** The Nominal overlap measure is derived in the same manner as the *ERI*, except for the difference that the portfolio overlap is *not* adjusted for the liquidity of the underlying assets:

$$\Omega_{Nominal} = \Pi \Pi^\top. \tag{18}$$

In analogy to the *ERI*, the Nominal overlap measure is defined as the Perron eigenvector of $\Omega_{Nominal}$.

By defifnition, these measures are also given by a vector of unit size, in which the value of the individual components can thus be interpreted as the corresponding bank's systemicness. We begin by analysing the correlation between the *ERI*, Size, Cosine Similarity and the Nominal

[6]However two caveats apply: Firstly, when banks liquidate assets in an order close that is close to the ranking from the most liquid to the least liquid (as normally $\delta_\mu >> \Pi^{i,\mu}$), it is not clear that the most liquid asset ex-ante should still turn out to be the most liquid ex-post. Including such considerations requires a dynamic model for the market depth, which is outside the scope of this paper. Secondly, When banks are constrained by risk-weighted capital, they have an incentive to liquidate assets with higher risk-weights first (Braouezec and Wagalath (2018)). As assets with higher risk weights usually are less liquid, this would incentivise banks to liquidate assets from the *least* liquid to the most.

	ERI	Nom. Ov.	Cos. Sim.	Size
ERI	1	**0.68** (0.85)	**-0.13** (- 0.22)	**0.60** (0.80)
Nom. Ov.		1	**-0.14** (-0.22)	**0.78** (0.92)
Cos. Sim.			1	**-0.17** (-0.27)
Size				1

Table 6: Similarity between the various overlap measures: The bold numbers are rank-correlations (Kendall's τ), while the numbers in brackets are linear correlations (Spearman's ρ).

Overlap Table 6. The bold numbers refer to rank-correlations (Kendall's τ) while the figures in parentheses refer to classical linear correlations (Spearman's ρ). There is a positive correlation between the ERI, the Nominal Overlap measure and the Size measure, while the correlation is negative between the Cosine Similarity and all other measures. Intuitively, we find the highest correlation between the Size and Nominal Overlap measures. While the ERI is positively correlated to these measures the correlation is lower (0.60- 0.68).

Figure 9 shows the systemicness ranking of the individual banks according to the four measures. The high correlation between the Nominal overlap and the Size measures is clearly visible in these graphs. All of the banks that are identified are either GSIBs or DSIBs, with the difference that the importance of banks such as Santander, ING or UniCredit decreases in the Nominal overlap measure relative to the others. In the ERI, the French banks Crédit Agricole (CA), BNP Paribas, Société Générale stand out, while UK banks, except for Barclays are significantly less systemic. The German Deutsche Bank is approximatively equally systemic according to all measures.

In contrast, the Cosine Similarity measure provides a completely different picture: The banks that dominate the first eigenvector of the Cosine Similarity matrix are "Erste Group Bank AG" (Erste), "Belfius Banque SA" (Bel), "KBC Group NV" (KBC), "Landesbank Hessen-Thüringen Girozentrale" (LHT), "Banco Popular Españ ol S.A." (BPE), "Banco de Sabadell S.A." (BS), "Groupe Crédit Mutuel" (GCM), "OTP Bank Nyrt." (OTP), "Intesa Sanpaolo S.p.A." (Int), "N.V. Bank Nederlandse Gemeenten" (BNG), "Svenska Handelsbanken - group" (SHB), "Swedbank - group" (SBG). The similarity measure thus allows to identify clusters of similarly composed portfolios that would be affected simultaneously by market downturns.[7] Whether portfolios are similarly composed (and thus exposed to the same types of market shocks) is a different question from whether portfolios can cause large price-mediated contagion to each other. While for the former issue size is indeed irrelevant, it is highly relevant for the latter. The Cosine Similarity measure thus captures a different type of information relating to portfolio overlaps.

As a final exercise in this subsection, we run stepwise regressions to analyse whether the similarity and nominal overlap measures improve the prediction of fire-sales losses over and above the ERI and size, as we did in Table 2. Table 7 shows that the Nominal overlap eigenvector performs relatively well compared to the ERI, while the Cosine Similarity eigenvector performs

[7]The second eigenvector of the matrix built from cosine similarity loads higher on the international banks, captured by the other measures.

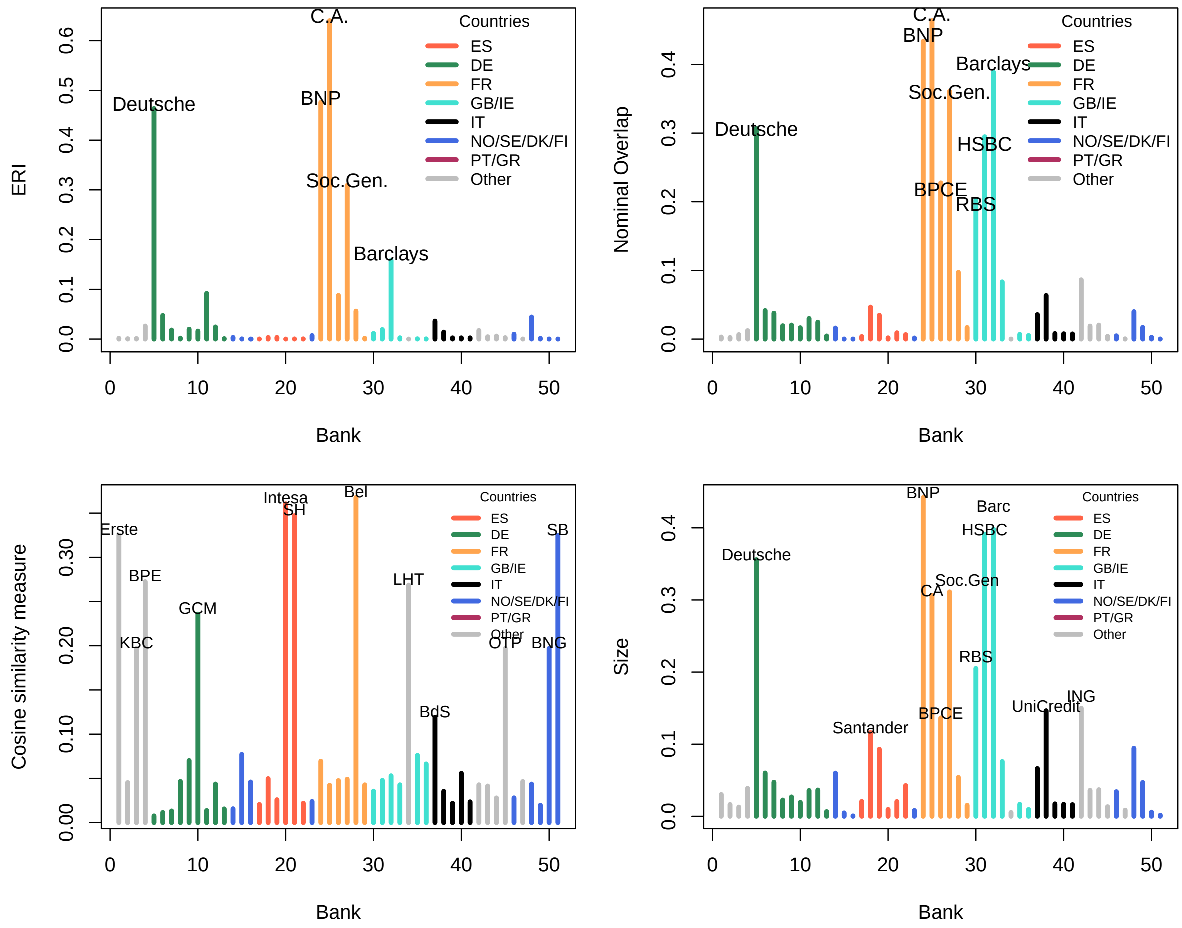

Figure 9: Various overlap and contagion measures. Endogenous Risk Index (top left), Nominal overlap (top right), Cosine Similarity (bottom left), size (bottom right).

worst in predicting fire-sales losses. Performing a model selection where we start with (i) no predictors, and (ii) all variables, leads us in both cases to Model 3, where Size and the ERI are the only final variables that are included, while the Nominal Overlap and Cosine Similarity are not.

When forcing all four indicators into the model, we see that the Nominal overlap and the Cosine Similarity measure can be discarded as they are not statistically significant. Finally, we perform this stepwise regression for all combinations of shock intensities and market depths in each of the four scenarios introduced earlier. When averaging across all of these parameters, the Size is included in 96.51 % of the cases as predictor. Second comes the ERI, which is included in 91.92% of the cases. The nominal overlap ERI is included in 51.96% of the cases, while the cosine similarity is only included in 6.54% of the cases.[8] Figures 20, 21, 22 and 23 in the appendix, from which we have computed the above percentages, show the inclusion/exclusion of the individual measures across all scenarios for all market impact & shock intensity combinations.

Finally, we run the stepwise regression model for all scenarios and shock size/market-impact combinations, assuming that banks *liquidate optimally*. In this case we find that the Size variable

[8]If in (13), one defines size starting from the total assets, rather than the liquid portfolio, Π only, results change a little. In this case the different variables are included as follows ERI 94.74%, Size 65.77% (significant drop!), Nominal overlap ERI 34.84%, while Similarity is never included.

Table 7: Size and ERI are retained as predictors for fire-sales losses.

	Model 1	Model 2	**Model 3**	Model 4
Size	–	–	0.58 ($p = 0.0000$)	0.72 ($p = 0.0000$)
ERI	–	–	0.27 ($p = 0.0000$)	0.31 ($p = 0.0000$)
Nom. Ov.	0.59 ($p = 0.0000$)	–	–	-0.14 ($p = 0.1946$)
Cos. Sim.	–	-0.47 ($p = 0.0326$)	–	-0.01 ($p = 0.9177$)
R^2	0.70	0.09	0.86	0.86
RMSE	0.38	0.66	0.27	0.27
n	51	51	51	51

is included in 78.54% of the cases, the ERI in 42.10 % of the cases, the Nominal Overlap in 64.11 % of the cases, while the Cosine Similarity measure is included in less than 0.1 % of the cases. In this particular case, the Nominal Overlap measure has on average a higher predictive value than the Endogenous Risk Index. This has an intuitive reason: liquidating more liquid assets first, obviously leads to larger sales and (relatively speaking) larger impacts in the more liquid asset classes, which to some extent counters the liquidity-weighting that enters the ERI.

3.6 Quantifying interconnectedness for GSIBs and SIFIs

Given the robust performance of the ERI, we propose in this final section how our indicators could be used for quantifying the interconnectedness between systemically important financial institutions and Global Systemically Important Banks in particular.

First, we compare the ranking of European banks according to the Indirect Contagion Indicator (ICI) and the Endogenous Risk Indicator respectively. Recall that the Indirect Contagion Index is the Perron eigenvector of $\Omega_0 := \Omega - \text{diag}(\Omega_{11}, \ldots, \Omega_{NN})$, which thus disregards self-inflicted contagion and solely accounts for contagion to *other* portfolios. Figure 10 shows the ICI (coloured bars) and the ERI (black crosses) for the largest European banks. The most systemic banks, according to our analysis of the 2016 public EBA data, are Crédit Agricole, Deutsche Bank and BNP Paribas, as well as on a slightly lower tier, Société Générale and Barclays. The main difference between the ERI and ICI ranking of the banks is that, relative to the ERI, Barclays and Société Générale become more important, while Crédit Agricole becomes less significant. Ranking a bank via the ERI versus the ICI boils down to the question whether one would like to focus on systemic externalities to other market participants only, or also include self-contagion.

Next, we turn to the the Basel Committee on Banking Supervision's methodology to quantify the systemic importance of Global Systemically Important Banks which is summarized in Figure 11.[9] Depending on which total score a bank achieves in this rating, its Common Equity Tier 1 (CET1) capital ratio requirement increases in 0.5% steps from 1% to 3.5%. This buffer is called the "additional loss absorbency requirement" or sometimes less formally the "GSIB buffer".[10] Currently, "interconnectedness", which contributes 20% to the overall score, is only assessed through total intra-financial system assets and liabilities as well as the amount of securities

[9]Basel Committee on Banking Supervision, Global Systemically Important Banks: Updated Assessment Methodology and the Higher Loss Absorbency Requirement. URL: `https://www.bis.org/publ/bcbs255.htm`

[10]See: `https://www.bis.org/bcbs/gsib/cutoff.htm`.

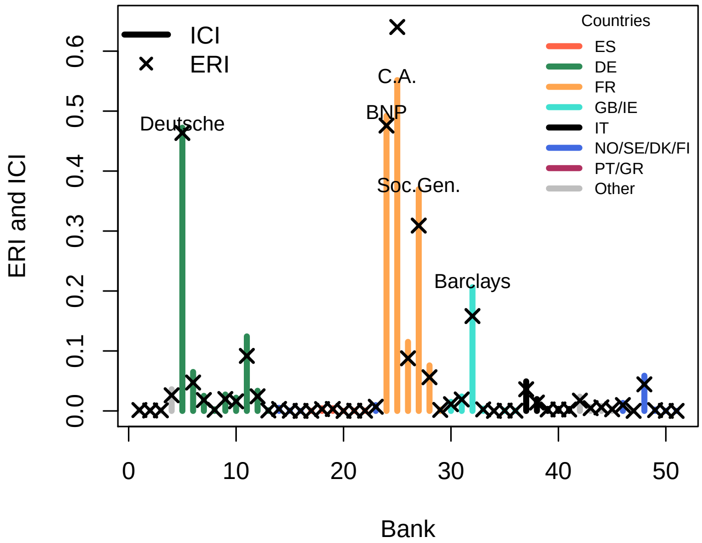

Figure 10: ICI which discounts losses inflicted on the own portfolio relative to the losses inflicted on other portfolios, and the ERI (black crosses) for the European banking system. Data source: EBA 2016, Calculations: Authors.

outstanding. This quantities only depend on the size of banks' securities holdings and are insensitive to *how* portfolios overlap. Moreover, as Figure 12 shows, these measures are closely related to the total size of the banks' balance sheet (captured by the "Size" component of the BCBS methodology.

For these reasons, we suggest that computing the Endogenous Risk Indicator or the Indirect Contagion Indicator for the most important global financial institutions would reveal more valuable information on their interconnectedness than the size of their securities holdings alone. A rough path to how such a monitoring could be set up is sketched in the Conclusions below.

4 Conclusion

Measures of endogenous risk and indirect contagion. Starting from a model to quantify fire-sales losses in scenarios of distressed portfolio liquidations, we derive two micro-founded measures to monitor indirect contagion: the Endogenous Risk Index (ERI) and the Indirect Contagion Index (ICI). The ERI, which is defined as the Perron eigenvector of the matrix of liquidity-weighted portfolio overlaps was shown to be a good proxy for fire-sales losses, possess an intuitive interpretation, and outperforms other measures of contagion in pinpointing the most systemic institutions across a wide range of different modeling assumptions and parameters. We have further suggested how the ERI can be modified to discount self-inflicted fire-sales losses and focus solely on spillover losses inflicted on other institutions, the ICI.

Global monitoring of overlapping portfolios. In Section 3.6, we suggested that the ERI and/or the ICI can improve the quantification of interconnectedness of global systemically important banks beyond current measures. Firstly, within jurisdictions, such as the EU or the US, these measures can be readily computed at a high level of precision using new and granular

Indicator-based measurement approach Table 1

Category (and weighting)	Individual indicator	Indicator weighting
Cross-jurisdictional activity (20%)	Cross-jurisdictional claims	10%
	Cross-jurisdictional liabilities	10%
Size (20%)	Total exposures as defined for use in the Basel III leverage ratio	20%
Interconnectedness (20%)	Intra-financial system assets	6.67%
	Intra-financial system liabilities	6.67%
	Securities outstanding	6.67%
Substitutability/financial institution infrastructure (20%)	Assets under custody	6.67%
	Payments activity	6.67%
	Underwritten transactions in debt and equity markets	6.67%
Complexity (20%)	Notional amount of over-the-counter (OTC) derivatives	6.67%
	Level 3 assets	6.67%
	Trading and available-for-sale securities	6.67%

Figure 11: BCBS GSIB Indicator measurement approach. Source: Basel Committee on Banking Supervision (2013).

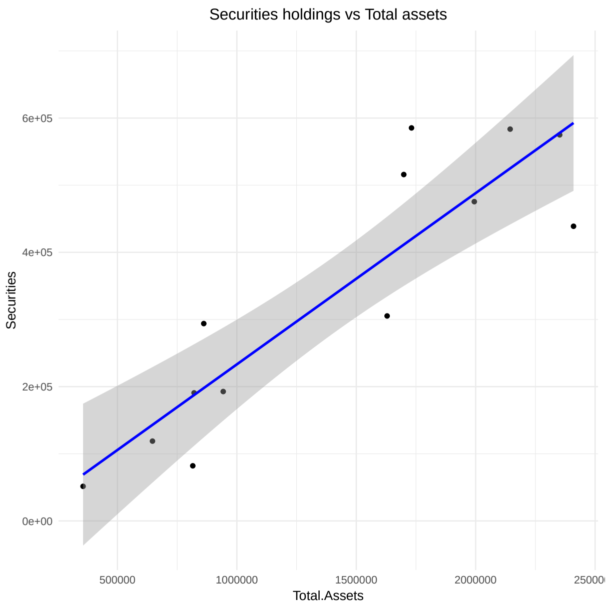

Figure 12: Strong correlation between total assets and securities holdings of the largest global banks. Amounts taken from annual reports 2015. ($R^2 = 0.8$, slope $= 0.255$, p-value $= 3.5 \times 10^{-5}$).

regulatory datasets such as the Securities Holdings Statistics (SHS) in Europe.

Secondly, while on a global level the need to build the full portfolio matrix Π of asset holdings raises some questions regarding data confidentiality, these are surmountable. We will sketch a rough roadmap, how this could be implemented below: First, one requires unique institutional identifiers (e.g. LEIs) and unique security identifiers (e.g. ISIN), a matter on which significant progress has been made over the recent years, see e.g. Committee on Payments and Market Infrastructures & International Organization of Securities Commissions (2017). The most important regulatory authorities can then upload data pertaining to the most important institutions of their respective jurisdiction to an encrypted server, which would be hosted by an international and independent organisation, as for instance the BIS or the IMF. If data is properly matched using unique institutional and product identifiers, the matching and construction of the portfolio matrix Π can be automated such that no jurisdiction, nor any individual at the host institution requires access to the data. The estimation of market depths does not involve confidential data and can thus be performed separately. Finally, the computation of Ω, the ERI and the ICI can be performed automatically on the server, and all data is deleted once the computation is completed. The ICI and ERI would then allow to quantify the most important overlaps on a global level, while respecting data confidentiality.

As a final remark, this approach is not limited to banks: as price-mediated contagion can also cause spillover losses between different sectors (e.g. from the asset management sector as in (Calimani et al., 2016)), this methodology would allows to capture the amount of interconnection and potential contagion between different sectors of the financial market.

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A Appendix

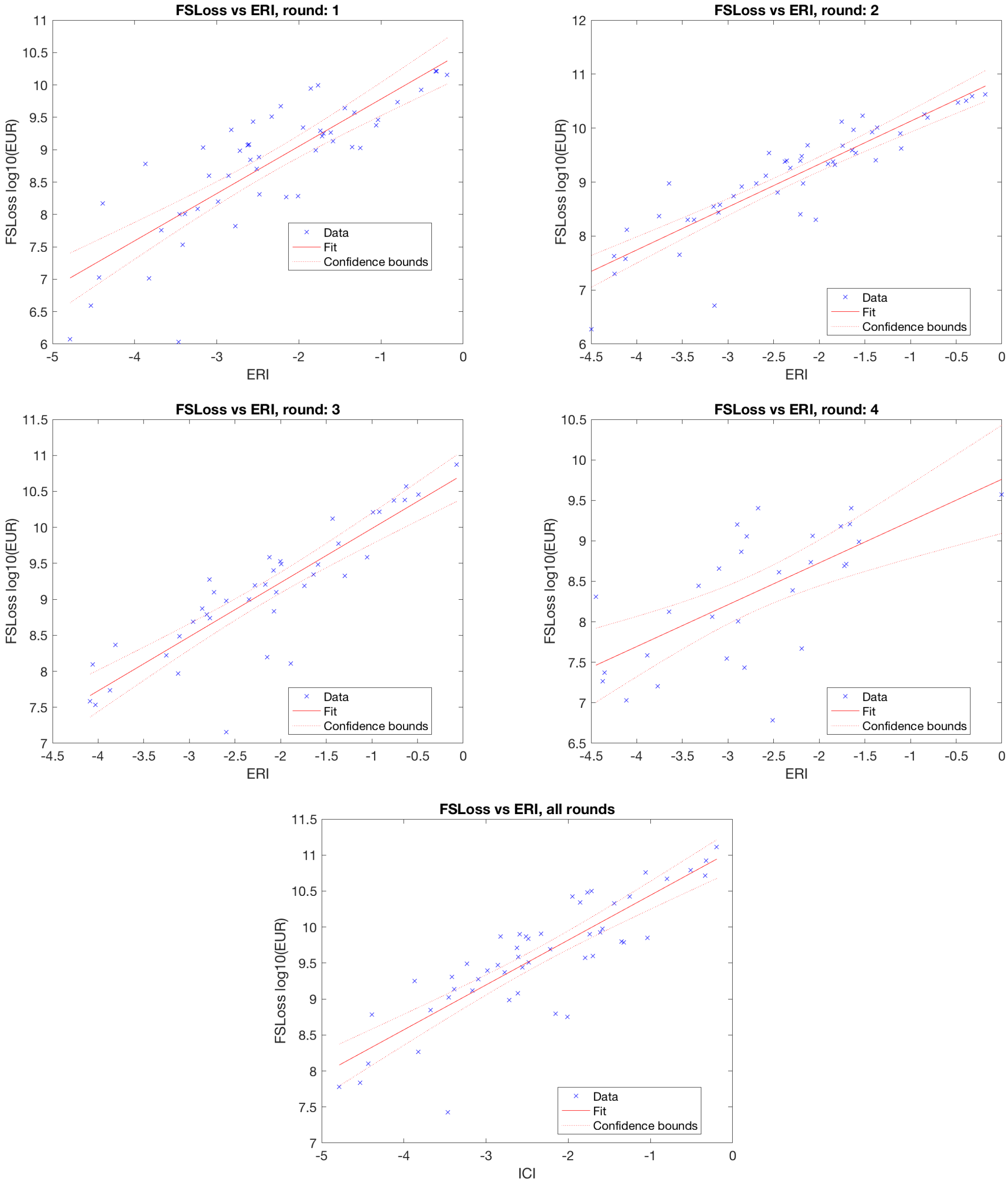


Figure 13: Bank-level fire-sales losses regressed on the ERI under proportional deleveraging.

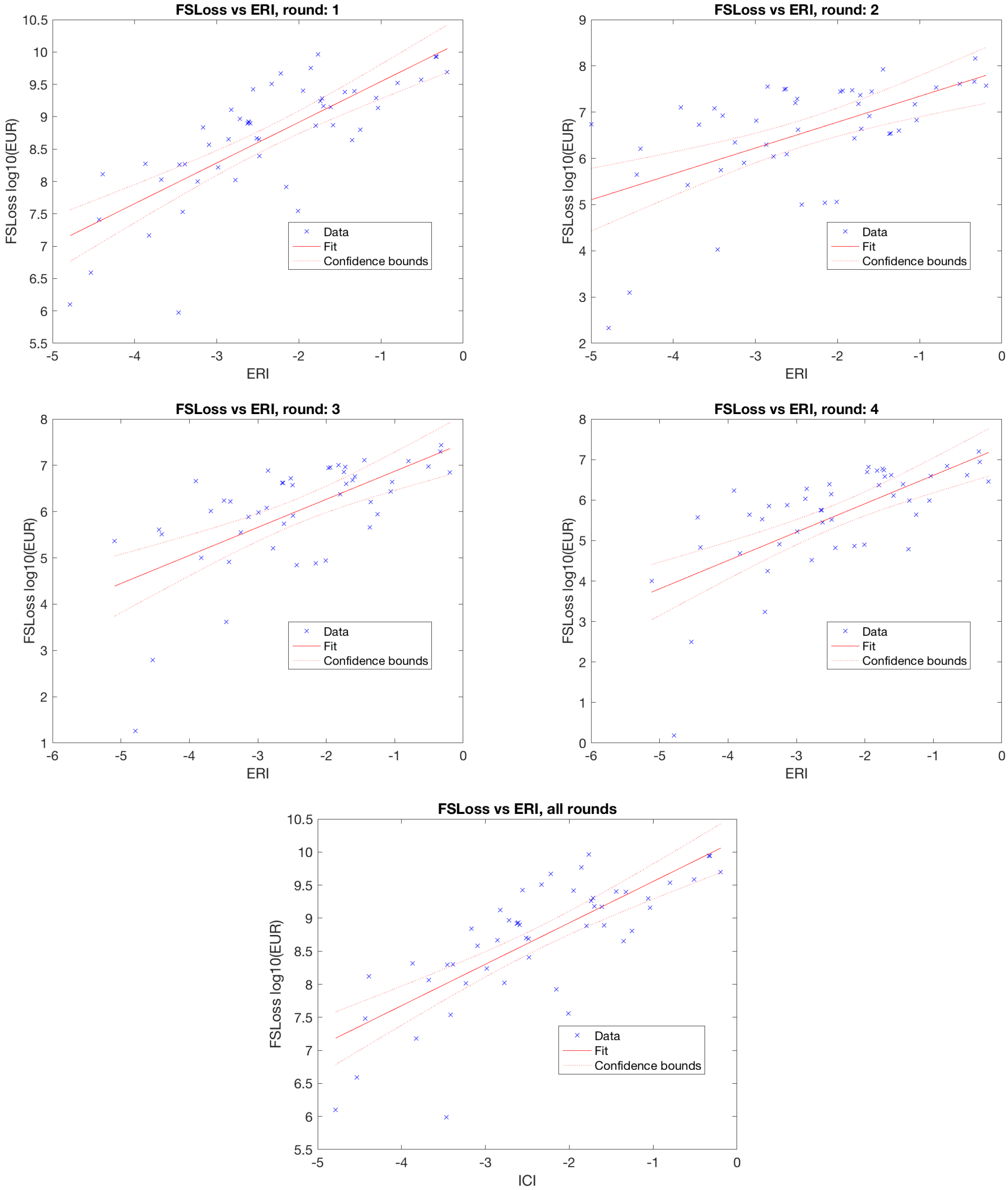


Figure 14: Bank-level fire-sales losses regressed on the ERI under optimised deleveraging.

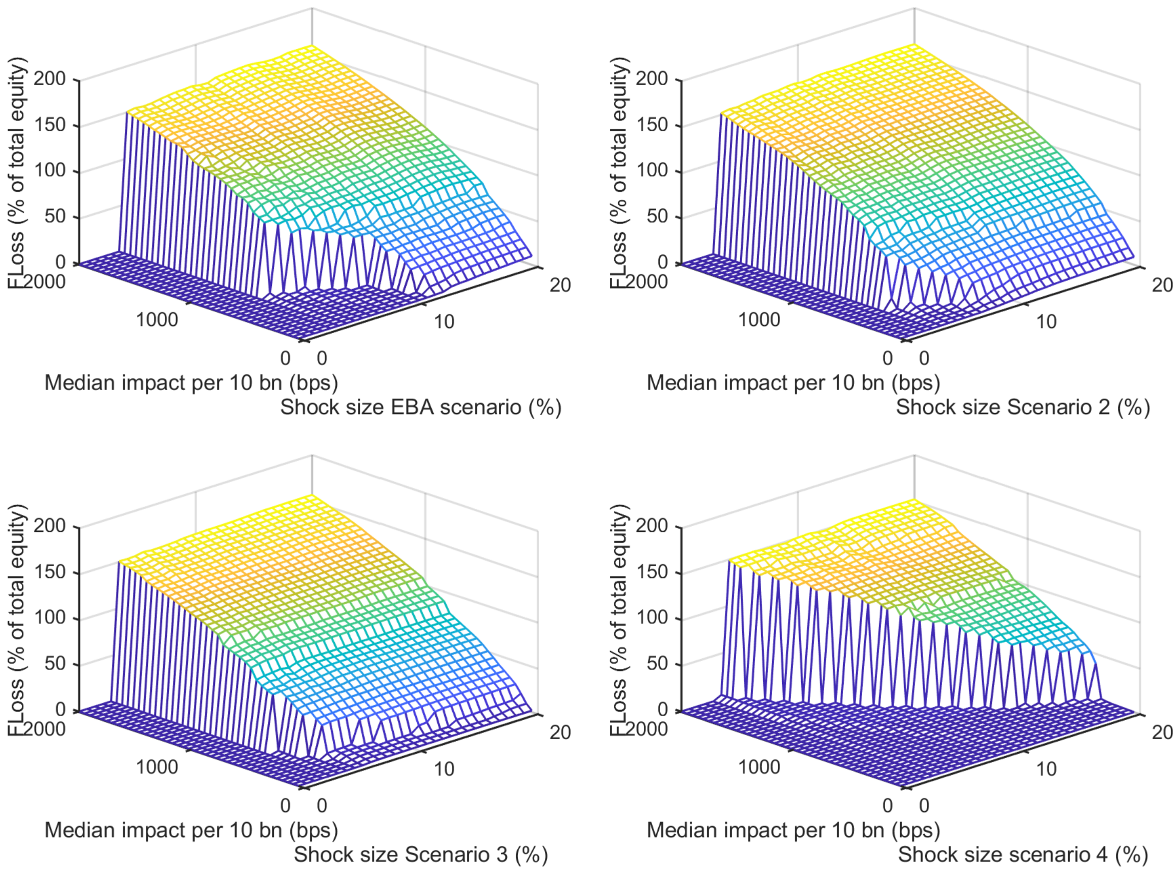


Figure 15: Fire-sales losses as a function of the shock intensity and the market-impact for the four scenarios.

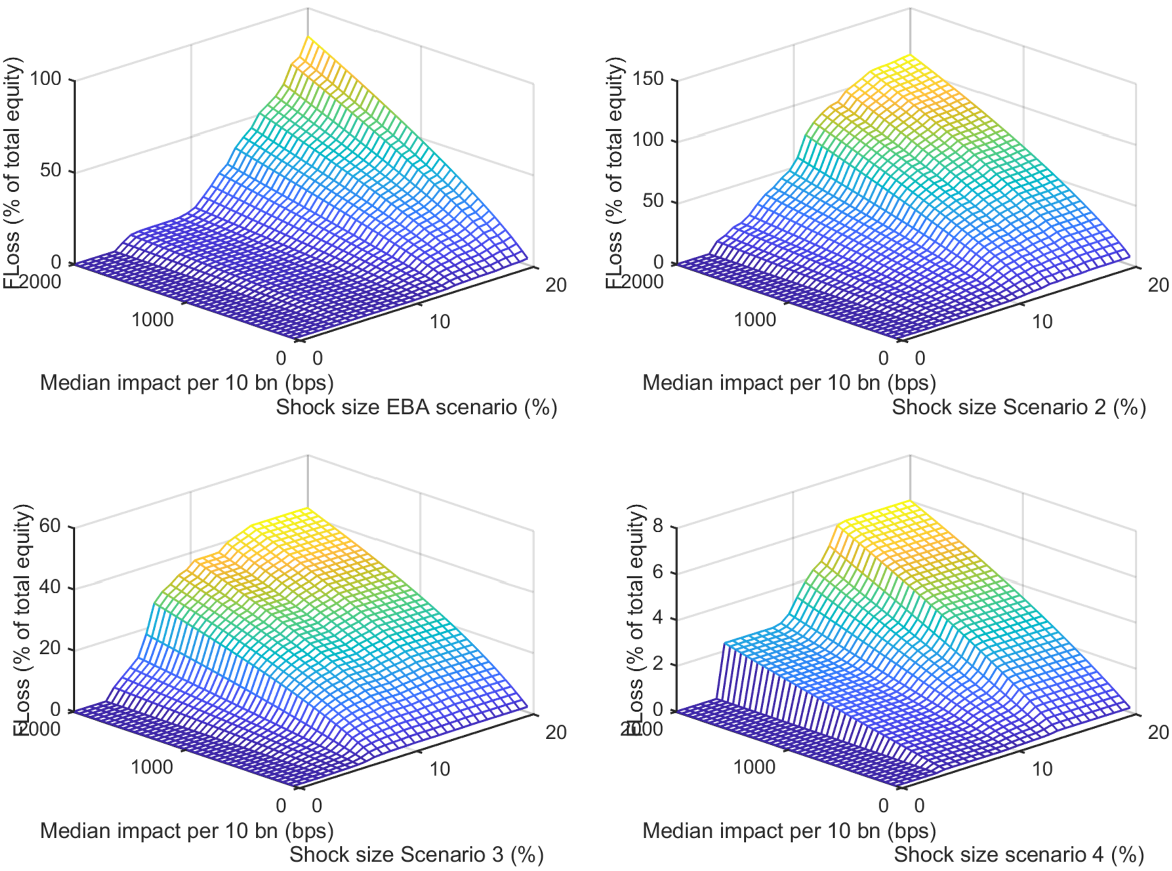


Figure 16: Fire-sales losses as a function of the shock intensity and the market impact for the four scenarios under optimized deleveraging (note the different z-axis scales!).

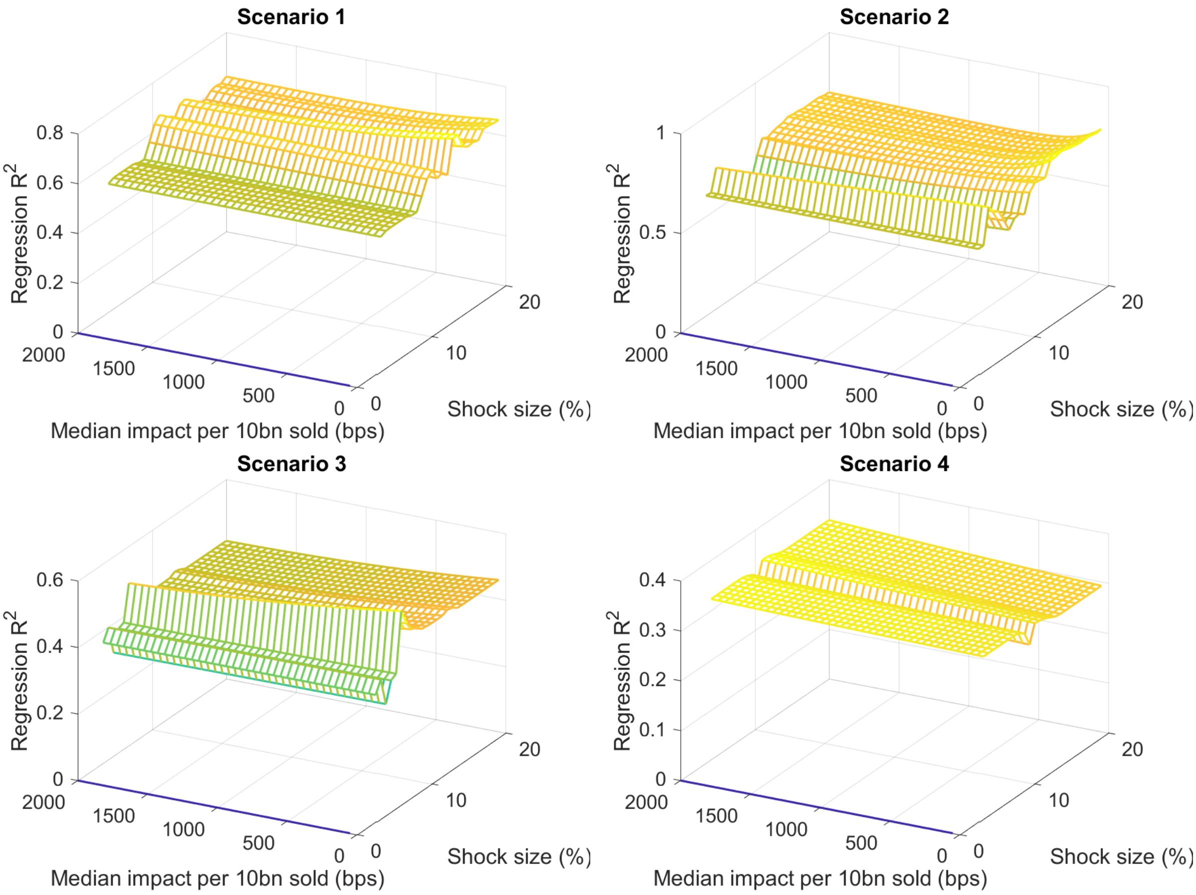


Figure 17: R^2 from regressing the fire-sales losses on the ERI in the optimal deleveraging case for all four scenarios and combinations of market impact and shock intensity.

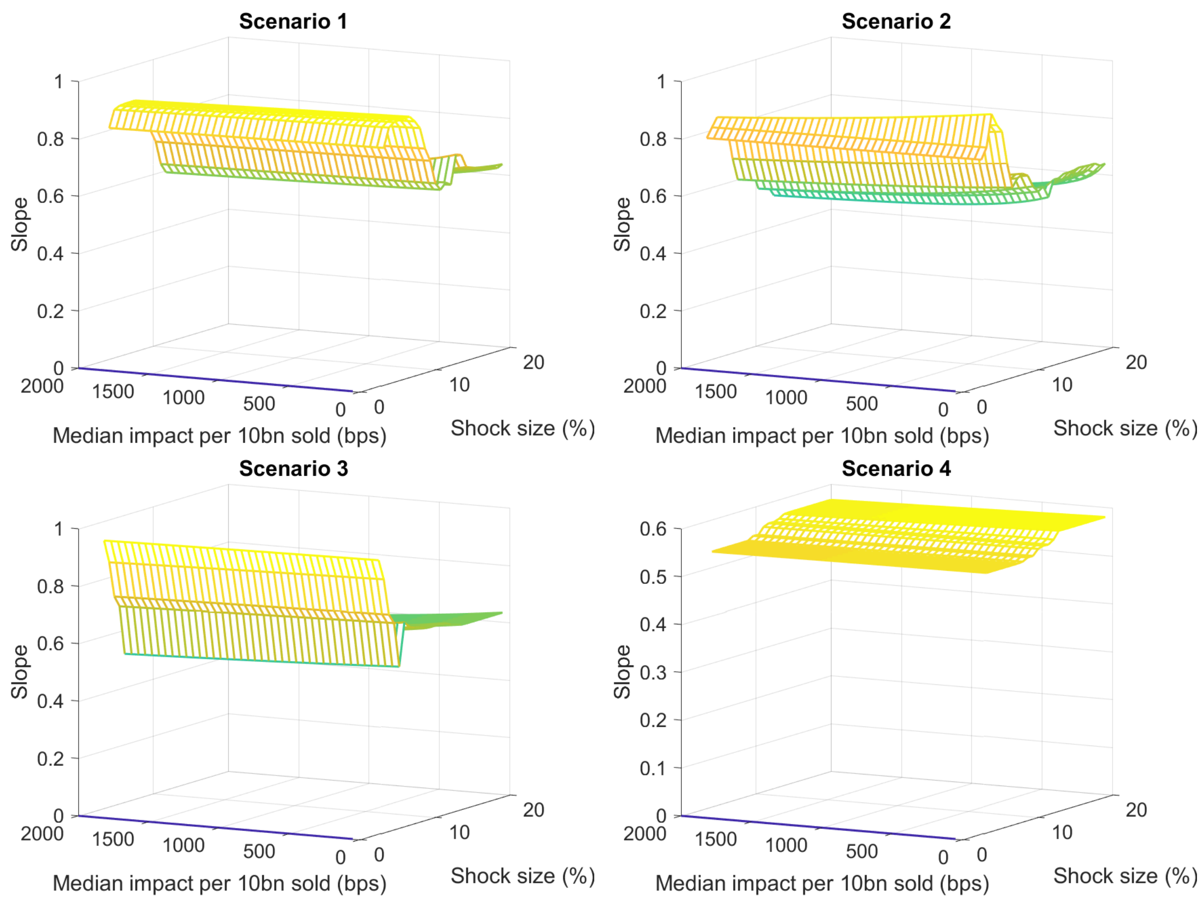


Figure 18: Slopes of the regression of the fire-sales losses on the ERI under optimised deleveraging.

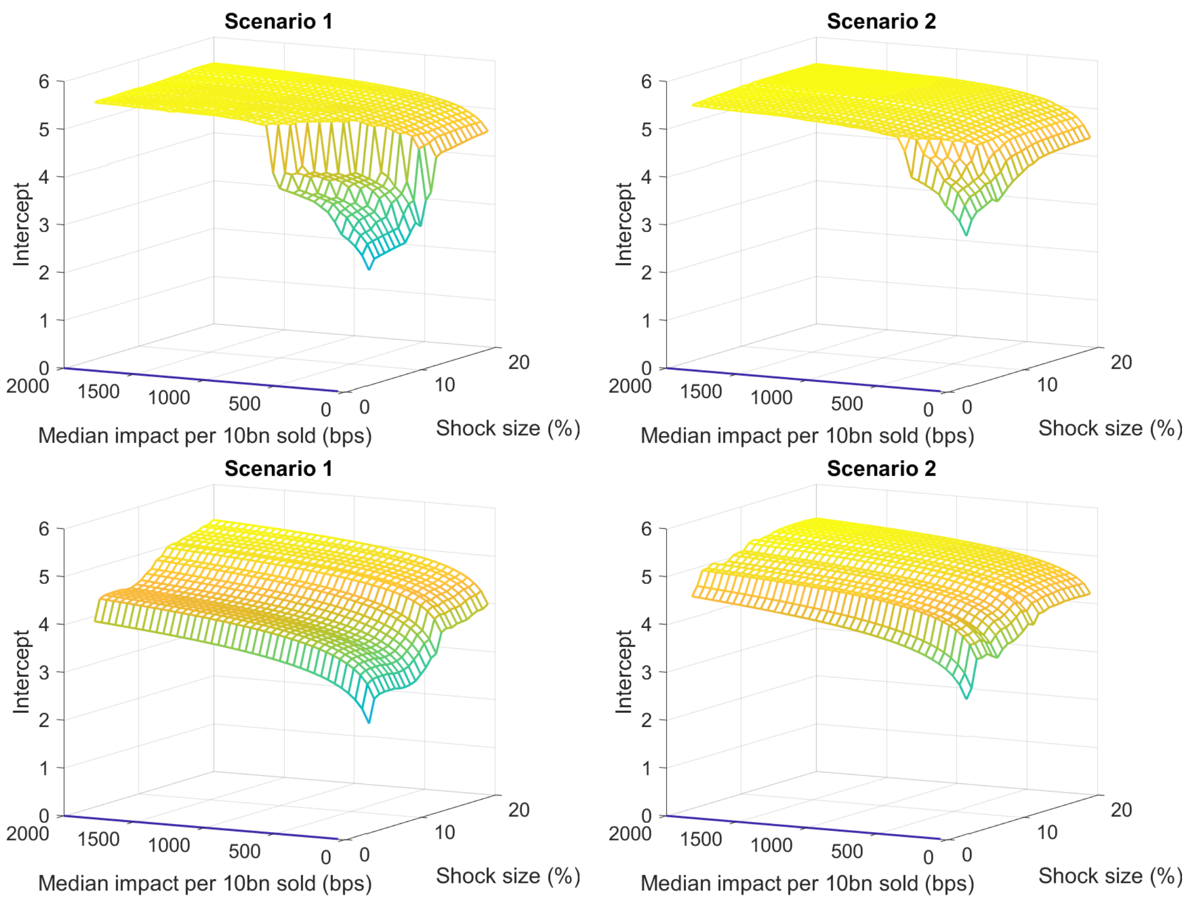


Figure 19: Intercepts of the regression of fire-sales losses on ERI for scenarios 1 and 2 under proportional deleveraging (top) and optimised deleveraging (bottom).

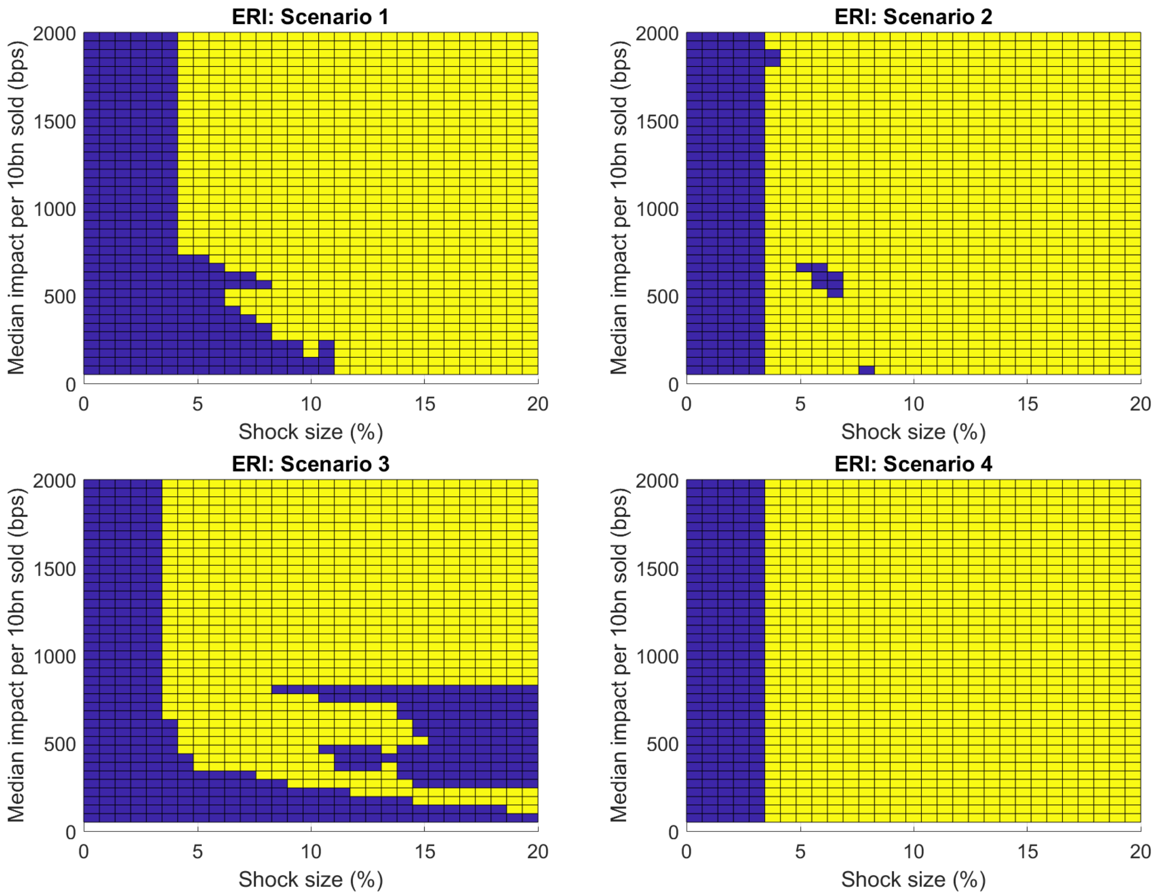


Figure 20: Inclusion (yellow) or exclusion (blue) of the ERI as a predictor for fire-sales losses in a stepwise regression model for all scenarios and market impact & shock size combinations.

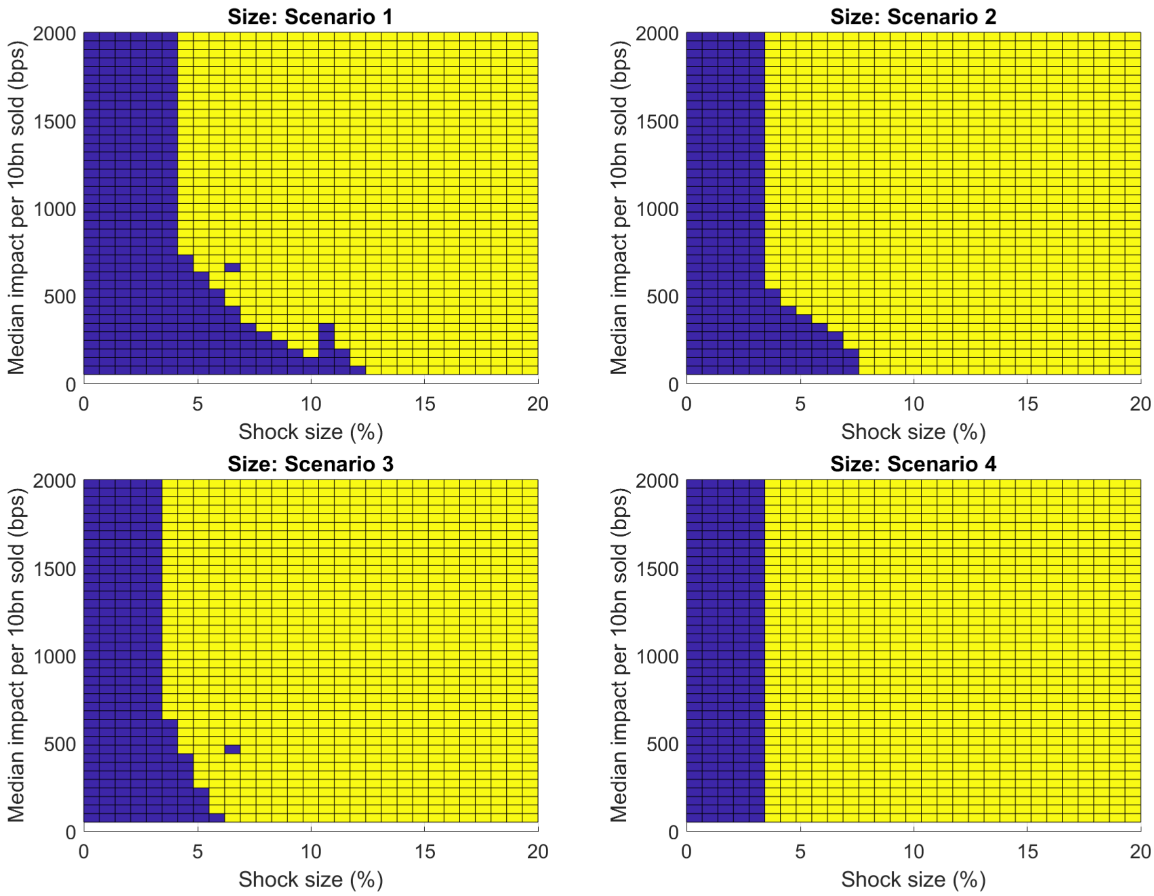


Figure 21: Inclusion (yellow) or exclusion (blue) of the Size as a predictor for fire-sales losses in a stepwise regression model for all scenarios and market impact & shock size combinations.

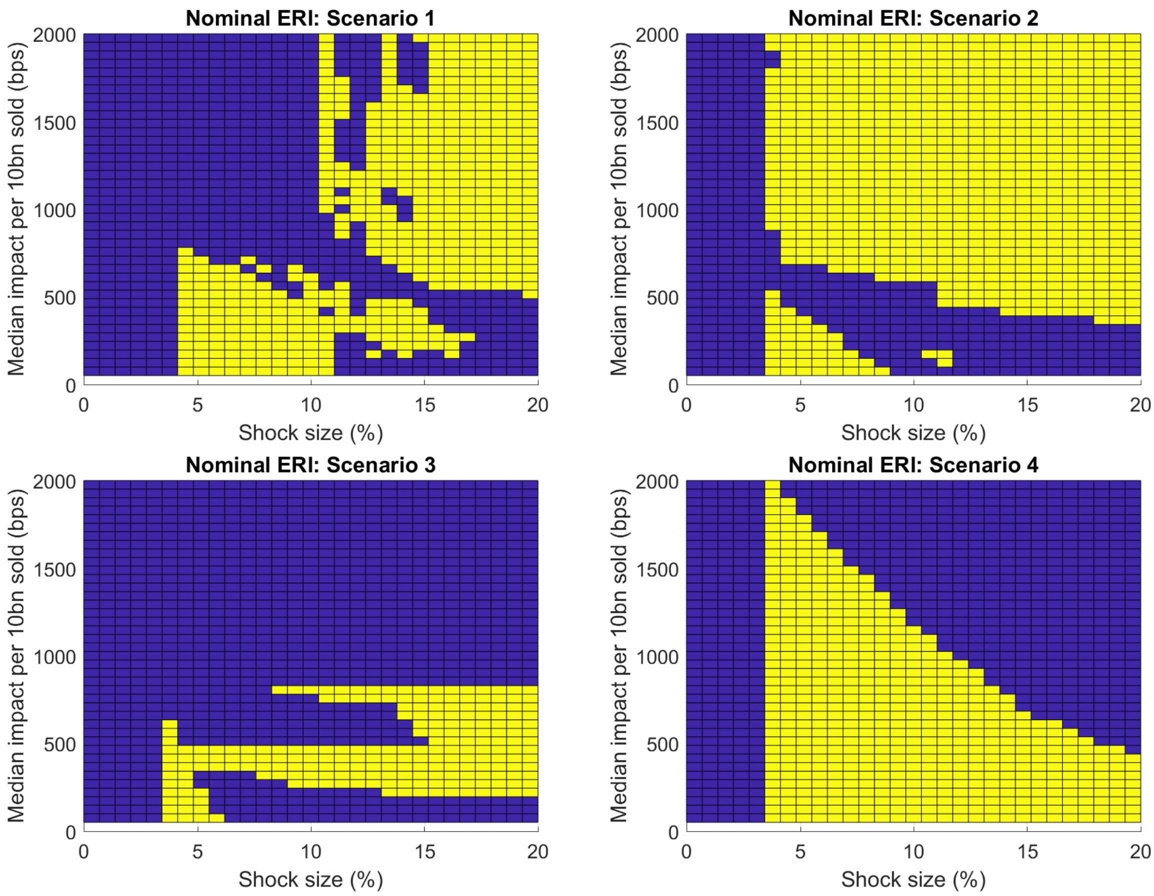


Figure 22: Inclusion (yellow) or exclusion (blue) of the Nominal-ERI as a predictor for fire-sales losses in a stepwise regression model for all scenarios and market impact & shock size combinations.

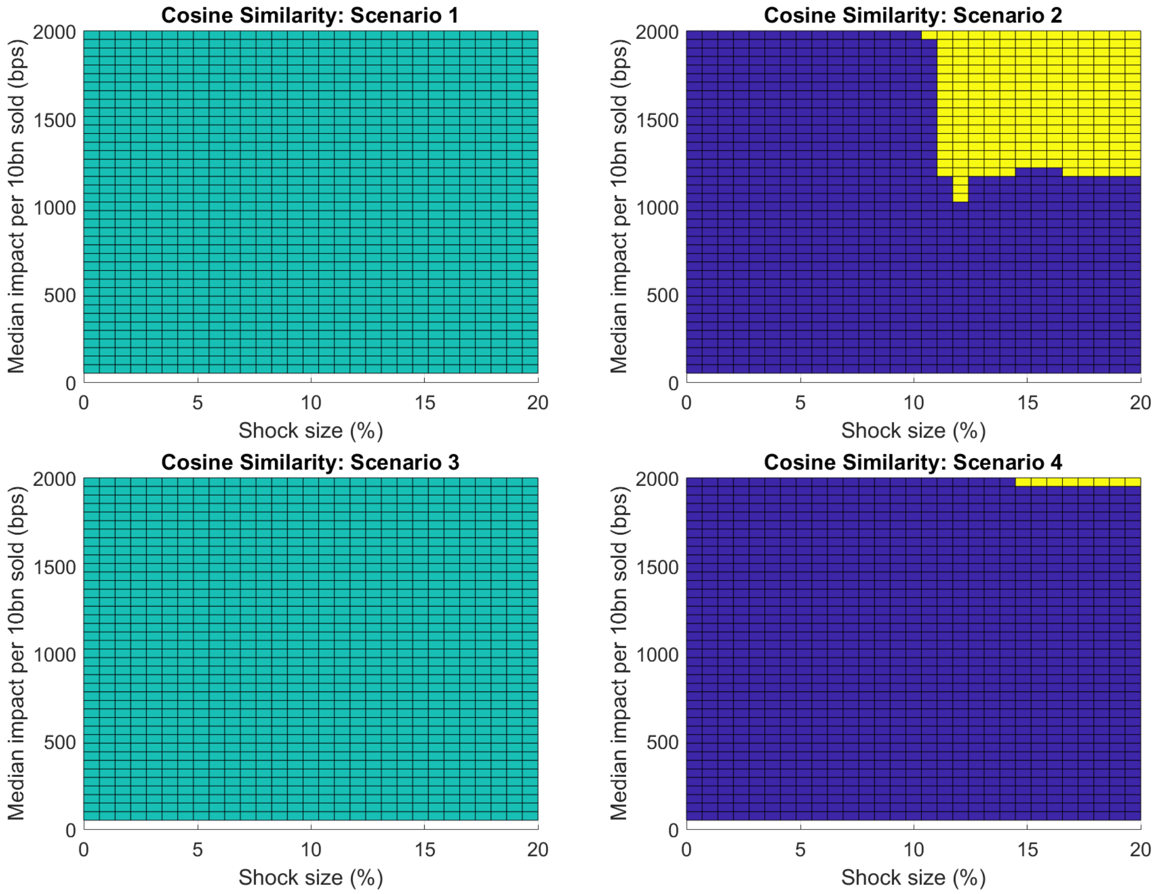
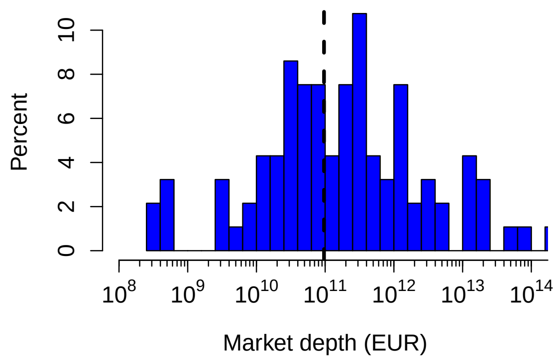


Figure 23: Inclusion (yellow) or exclusion (blue) of the Cosine Similarity as a predictor for fire-sales losses in a stepwise regression model for all scenarios and market impact & shock size combinations.



A.1 EBA data codes

Assets	**EBA Item**	**EBA Exposure**
Illiquid assets Θ		
Residential exposures ($\epsilon_\kappa \geq 0$)	1690201, 1690301	4120, 4320, 4700, 5000
Commercial exposures ($\epsilon_\kappa \geq 0$)	1690201, 1690301	3100, 3200, 4110, 4200 4310, 4500, 6600
Marketable assets Π		
Corporate exposures	1690201, 1690301	1100, 1200, 1300, 1400 1500, 1700, 2100, 2200 3000, 6100, 6200, 6500
Sovereign exposures	1690503, 1690506 1690507, 1690508	not applicable
Direct sovereign exposures in derivatives	1690511	not applicable

Liabilities	**EBA Item**	**EBA Exposure**
Tier 1 capital	1690110	not applicable
Deposits and debt	-	-

Other information	**EBA Item**	**EBA Exposure**
Total loans	1690903	not applicable
Total marketable assets	1690902	not applicable
Total leverage ratio exposure	1690111	not applicable

Table 8: Mapping of EBA data to model variables.