When Unity Makes Strenght: A Systemic Risk Index*

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Abstract: Due to the recent financial crisis, several systemic risk measures have been proposed in the literature for quantifying financial system-wide distress. In this note we propose an aggregated Index for financial systemic risk measurement based on Principal Component Analysis on the several systemic risk measures released in the recent literature. We use this index to further identify the states of the market as suggested in Boucher et al. [2014]. We show, by characterizing markets conditions with a robust Kohonen Self-Organizing Maps algorithm that this measure is directly linked to crises markets states and there is a strong link between return and systemic risk.

1 Introduction

Following the experience of the recent 2007-2009 financial crisis, a special attention has been paid to the "macroprudential" regulation, *i.e.* the prevention of a financial system-wide distress that can adversely impact the real economy. For this purpose, several systemic risk measures have been developed since 2010. Some of the most important described in Bisias et al. [2012] and Giglio et al. [2013] include the Conditional Value-at-Risk (CoVaR), the Delta Conditional Value-at-Risk (ΔCoVaR) and the Marginal Expected Shortfall (MES). Other important measures are the SRISK from Brownlees and Engle [2012] and the Component Expected Shortfall (CES) from Banulescu and Dumitrescu [2012].

However, these measures are focused on different aspects of financial risk, such as leverage, kurtosis or skewness, and thereby the comparison of two firms may present dissimilar results according to the chosen measure.

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In this paper, following the intuition of Giglio et al. [2013], we aim to create an aggregated index able to identify the main systemic risk factors through the study of 16 systemic risk measures applied to American securities.

We will first try to identify the link between this index and the market states by using the neural network classification algorithm corresponding to the Self-Organizing Maps (SOM) in its robust version of Guinot et al. [6], known as R-SOM. We will then apply a Principal Component Analysis on the selected measures, which will allow us to establish an index based on the so-called Principal Components and their respective weights. Then, following the same intuition, we construct the aggregated index by applying an Independent Component Analysis. Our results confirm the findings in Giglio et al. [2013] that when the measures are studied as a whole, they contain important predictive information about future macroeconomic outcomes.

2. Identifying the Link between Markets States and the Index of Systemic Risk Measures

The aim of the classification by robust Kohonen maps is to identify the correlation between the Index of Systemic Risk Measures and the shape of the financial market.

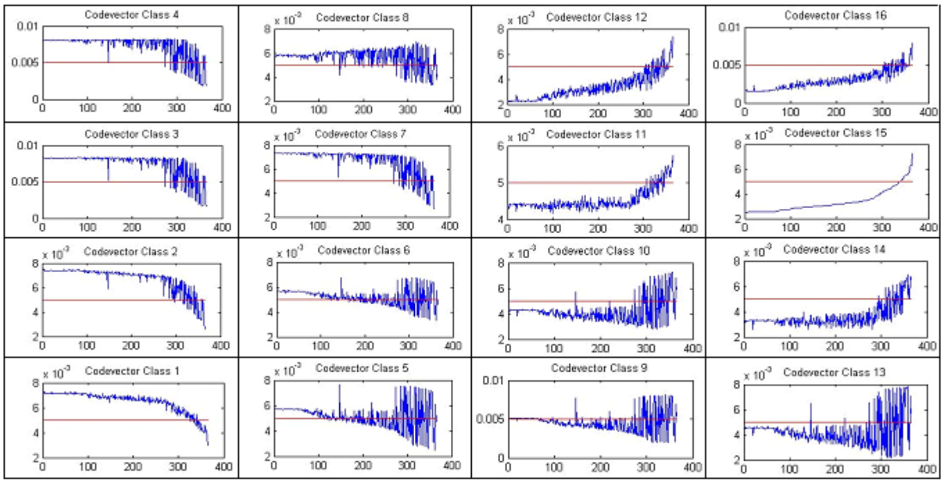
3.1 R-SOM Algorithm and MSCI US

In this part, we aim at analyzing a data set composed of 95 financial institutions in the American Stock Market, each of which are measured by 6 systemic risk measures (VaR95, CoVaR95, MES95, CES95, DeltaCoVaR95, SRISK95), evaluated on a daily basis from the 24nd of March, 2004 to the 31st of December 2010, that is 2724 dates. The size of the initial matrix M is thereby (2724 x 16).

We use for understanding the markets states the R-SOM algorithm proposed by Guinot et al. [6]. This approach provides a two-step stochastic method based on a bootstrap process to increase the reliability of the underlying neighbourhood structure. The increase in robustness is relative to the sensitivities of the output to the sampling method (see Guinot et al. [6], Kouontchou et al. [8], Olteanu et al. [9] and Sorjamaa et al. [10]).

Figure 1 shows the codevectors of the [4x4] systemic risk classes of the R-SOM algorithm, and the average line of value of each class in order to compare the values of these codevectors. Each class represents different dates for systemic risk measures of financial institutions.

Figure 1: Codevectors of the Systemic Risk Classes obtained
with the R-SOM Algorithm



We compute the performance of the MSCI US Index on the periods corresponding to the different systemic risk classes obtained with the R-SOM algorithm. In order to have a relevant idea of the evolution of the MSCI US, we only consider classes which contain at least 5% of the initial samples (*i.e.* classes that contain more than 86 samples dates).

Figure 2: MSCI US Performance over the periods corresponding
to the Systemic Risk classes obtained with the R-SOM Algorithm

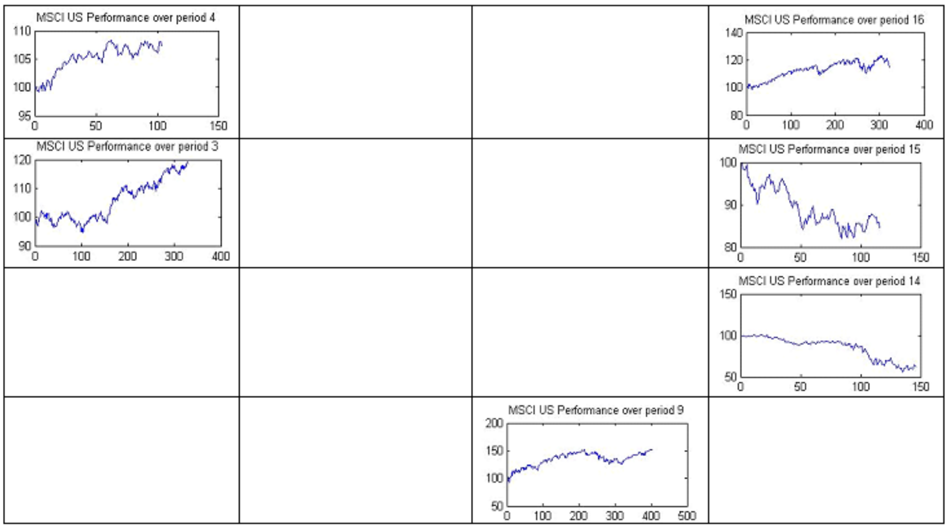
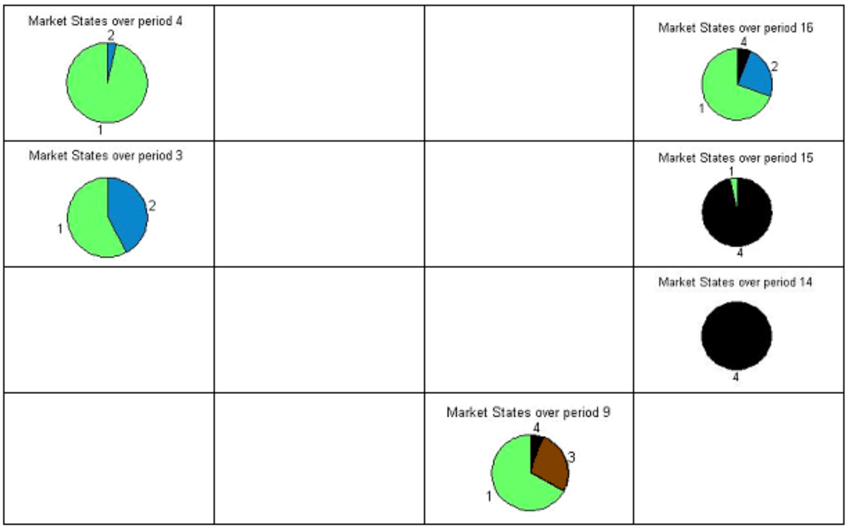


Figure 2 confirms that the MSCI US Index is significantly decreasing over the periods of classes 14 and 15, whereas it is increasing over the other periods. If we compare these results to the former figure, we note that classes 14 and 15 codevectors are lower than those of classes 3, 4, 9 and 16; this comparison thus underlines the link between the ISRM and the evolution of the MSCI: a fall of the MSCI Index is associated with a rise of the ISRM, here represented by a lowering of the codevectors of the R-SOM algorithm.

3.2 Characterizing Market Conditions with the ISRM

We now focus on the link between the ISRM through the R-SOM algorithm, and the market states defined above. With the same principle as in Figure 2, we represent on Figure 3 the part of each Market State for the 6 classes of the R-SOM algorithm that contain more than 5% of the initial samples.

Figure 3: Parts of the Market States over the Periods associated with the R-SOM Classes



In Figure 3, we clearly note that classes 14 and 15 are nearly exclusively composed of samples which dates are associated with Market State 4, whereas the other classes match more with States 1 and 2, which stand for periods of growth of the market.

Figure 2 and 3 thus confirm the correlation between the Index of Systemic Risk Measures and the State of the Financial Market: a decrease of the codevectors values is associated with a decrease of the MSCI, and an increase of the Market State. This correlation is also represented by Figure 4 below, with the rise of the ISRM during the period of crisis identified by Market State 4.

2 Empirical Orthogonal Functions

The Empirical Orthogonal Function (EOF) analysis is a decomposition of a data set in term of orthogonal basis functions which are determined from the data. This method is also known as the Principal Component Analysis (PCA), applied to a group of time series data. The EOF method is thus used to analyse the variability of a single field, and can allow afterwards to denoise the initial data.

If we consider a (n, p) Matrix M of initial data, corresponding in our case to p time series of systemic risk measures, the algorithm is constructed as follows: after removing the mean value from each of the time series (*i.e.* each column of M), The covariance matrix C of M is formed. The next step consists then in solving the eigenvalue problem:

$$CR = R\Lambda, \tag{1}$$

where R is a matrix of eigenvectors of covariance matrix C, and Λ is a diagonal matrix containing the eigenvalues of C.

Assuming that Λ is ordered according to the size of the eigenvalues, the eigenvectors associated to those eigenvalues are the EOF we are looking for. The algorithm also builds the Principal Components of M (PC), called as well the expansion coefficients, which correspond to the evolution in time of the EOFs, and are given by the formula:

$$a_j = M c_j, \tag{2}$$

where a_j is the principal component associated with the *j*-th EOF, and c_j is the *j*-th column of C (the *j*-th EOF).

These Principal Components are critical in order to identify the main factors that are responsible for the variance of the initial data set: Indeed the EOF can be interpreted as the modes of variability of the time series, and the Principal Components show how these modes evolve in time.

The first principal component thus accounts for as much variance as possible in our initial matrix, and then the following ones also have the highest possible variance, under the constraint of orthogonality. This part of the variability can be quantified, regarding the percentage of the trace of the eigenvalue matrix Λ attributable to the first eigenvalue λ_1, and then to the following eigenvalues λ_2, λ_3, *etc.*

Since the PCA methodology is based on second order moments only, it lacks information on higher order statistics.

A different technique for data analysis called Independent Component Analysis (ICA) takes into account higher order moments and exploits inherently non-Gaussian features of the data. While the goal in PCA is to minimize the reprojection error from compressed data, the goal of ICA is to minimize the statistical dependence between the basis vectors. Mathematically, this can be written as:

$$M = As, \tag{3}$$

where A is an unknown matrix called the mixing matrix and M, s are the two vectors representing the observed signals and source signals respectively. The objective is to recover the original signals, s, from only the observed vector of zero mean M. We obtain estimates for the sources by first obtaining the "unmixing matrix" W, where, $W = A^{-1}$. This enables an estimate, $\hat{s}$ of the independent sources to be obtained:

$$s = WM, \tag{4}$$

We seek to obtain a vector y that approximates s by estimating the unmixing matrix W. If the estimate of the unmixing matrix is accurate, we obtain a good approximation of the sources. Unlike PCA, the basis vectors in ICA are neither orthogonal nor ranked in order. In ICA, there is no order of magnitude associated with each component. In other words, there is no better or worst components. Also, there is no closed form expression to find W. Instead, many iterative algorithms have been proposed based on different search criteria. However, it has been shown that most of the criteria optimized by different ICA algorithms lead to similar or even identical algorithms.

For the following empirical analysis we work with a data set composed of 95 financial institutions in American Stocks Market, each of which are measured by 16 systemic risk measures (Herfindalh, Absorption Ratio, turbulence measure, Term spread, Default Yield spread, TED spread, Volatility, VaR95, CoVaR95, MES95, CES95, DeltaCoVaR95, SRISK95, DCI, AIM et Spillover index), evaluated on a daily basis from the 28th of March, 2003 to the 28th of June 2014, that is 2724 dates. The size of the matrix M is thereby (2724 x 16 x 95). Those measures at the individual firm level (*i.e.* MES, CoVaR), have been aggregated using equal weights for each firm.

We thus construct an aggregated Index, which we will name Index of Systemic Risk Measures (ISRM). In a first step, we calculate this index based on a Principal Component Analysis, where each of the components is weighted by the part of variance that it represents, *i.e.* by the percentage of the eigenvalue associated to the component. In a second step, following the same intuition, we calculate the index based on an Independent Component Analysis using the algorithm FastICA by Hyvärinen and Oja [2000]. Note that since there is no order of magnitude in the ICA methodology, the independent components are equally weighted for the construction of the index.

Figure 4 displays the results obtained with the two methodologies, along with the Market Crises (state 4 of the market).

Figure 4: Index of Systemic Risk Measures and Market States

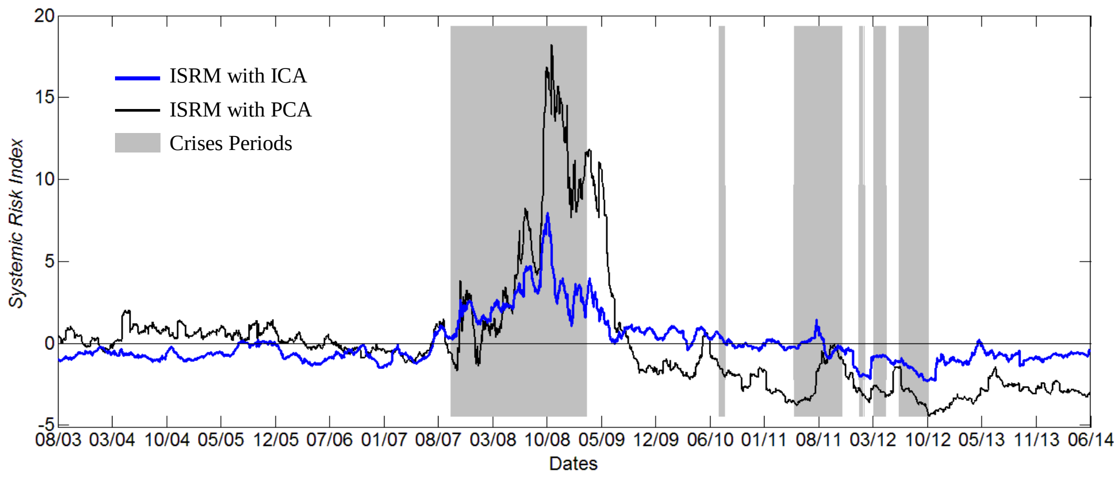


Figure 4 clearly shows a corresponding increase of the ISRM using both methodologies during the period corresponding to the state 4 of the market (crisis periods).

4 Conclusion

Through the analysis of different Systemic Risk Measures computed for different firms, we have been able to build an Index based on a mathematical algorithm relying on the variance between these different measures, which ties in the evolution of the financial market, as it has been confirmed by the simultaneity between the significant rise of our Index and the Market State 4, corresponding to the 2008 crisis. The comparison of the variations of the Index of Systemic Risk Measures and of the MSCI US also shows that they are globally opposite, which confirms the relevance of the Index towards the evolution of the financial market.

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Appendix A. Definitions of systemic risk measures

Absorption Ratio: proposed by Kritzman *et al.* (2010), this measure computes the fraction of return variance of a set of N financial institutions explained by the first K < N principal components:

$$AR = \frac{\sum_{i=1}^{n} \sigma_{Ei}^{2}}{\sum_{j=1}^{n} \sigma_{aj}^{2}} \tag{A.1}$$

where *n* is the number of eigenvectors, σ^2_{Ei} is the variance of eigenvector *i* and σ^2_{aj} is the variance of asset *j*.

AIM: proposed by Amihud (2002), AIM captures a weighted average of stock-level illiquidity *AIMit*, defined as:

$$AIM_t^i = \frac{1}{K} \sum_{\tau=t-K}^{t} \frac{|r_{i,\tau}|}{turnover_{i,\tau}} \tag{A.2}$$

Component Expected Shortfall (CES): proposed by Banulescu and Dumitrescu (2012), this risk measure based on the market expected shortfall (ES) measures the absolute contribution of the firm to the risk of the financial system (instead of the marginal contribution). It is calculated by calibrating the first derivative of the ES with the weights w_{it} defined for each financial institution:

$$\begin{aligned} CES_{it} &= [VaR_{it}(1-\alpha)] \\ &= w_{it} E[r_{it} \mid r_{mt} \leq VaR_{mt}(1-\alpha)] \end{aligned} \quad \text{(A.3)}$$

Co-VaR: proposed by Adrian and Brunnermeier (2011), corresponds to the VaR of the financial system, conditional to a systemic event observed in a financial institution. Formally, the CoVaR corresponds to a quantile of the returns of the market conditional to an event in a financial institution *i* such that:

$$\Pr ob[r_{mt} \leq CoVaR_{mt}(1-\alpha) \mid r_{it} \leq VaR_{it}(1-\alpha')] = \alpha \quad \text{(A.4)}$$

Co-CoVaR: the Co-CoVaR proposed by Boucher, Kouontchou, Maillet and Scaillet (2013), is a risk-model corrected version of the CoVaR. The CoVaR, based on quantile estimators, is very sensitive to extreme quantile measurement errors. Therefore it should take into account model risk.

$$Co - CoVaR_{mt|i=VaR_{it}(1-\alpha)}(1-\alpha) = \hat{\mu}_{\alpha}^{i} + \hat{\gamma}_{\alpha}^{i} VaR_{it}^{*}(1-\alpha) \quad \text{(A.5)}$$

Dynamic Causality Index (DCI): proposed by Billio *et al.* (2012) the DCI aims to capture how interconnected a set of financial institutions is by computing the fraction of significant Granger-causality relationships among their returns:

$$DCI_t = \frac{\#\, significant\, GC\, relations}{\#\, relations} \quad \text{(A.6)}$$

Expected Shortfall (ES): it is the expected return of those observations beyond the VaR at α%. It is defined as:

$$ES_{mt}(1-\alpha) = E[r_{mt} \mid r_{mt} \leq VaR_{mt}(1-\alpha)] \quad \text{(A.7)}$$

Herfindahl index: index that measures market concentration. This index captures the potential instability due to the threat of default of the most important companies:

$$Herfindahl_t = N \frac{\sum_{i=1}^{N} ME_i^2}{\left(\sum_{i=1}^{N} ME_i\right)^2} \quad \text{(A.8)}$$

where ME_i is the market equity of firm *i* and N is the number of firms.

Marginal Expected Shortfall (MES): the MES proposed by Acharya et al. (2010) is defined as the marginal contribution of a firm *i* to systemic risk, defined by the Expected Shortfall (ES). The MES is the partial derivative of ES on the weight of firm *i* in the economy.

$$MES_{it}(1-\alpha) = E[r_{it} \,|\, r_{mt} \le VaR_{mt}(1-\alpha)] \tag{A.9}$$

where r_{it} is the return of firm *i* in time *t* and r_{mt} is the return of the market in time *t*.

The MES can be historical, or calculated according to the methodology of Brownlees and Engle (2011) with dynamic volatility models.

Spillover Index: this index proposed by Diebold and Yilmaz (2009) aggregates the contribution of each variable to the forecast error variance of other variables (spillovers) across multiple return series. It captures the total extent of spillover across the series considered. It is a measure of interdependence that is expressed, in the case of a VAR of order *p* of *N* variables with a forecast of *H* periods, as follows:

$$S = \frac{\sum_{h=0}^{H-1} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} a_{h,ij}^2}{\sum_{h=0}^{H-1} trace(A_h A_h^{'})} \times 100 \tag{A.10}$$

where the numerator represents the total spillovers, and the denominator represents the total forecast error variation.

Systemic RISK Measure (SRISK): it is an extension of the MES to take into account for the debt and the size of the firm. It measures the expected capital shortfall of a financial institution, conditional on a crisis affecting the financial system as a whole.

$$\begin{aligned} &SRISK_{it}(1-\alpha) \\ &= \max\left\{0, \gamma D_{it} - (1-\gamma) w_{it}[1 - LRMES_{it}(1-\alpha)]\right\} \end{aligned} \tag{A.11}$$

with γ the prudential capital ratio, *D* the quarterly book value of total liabilities , w the market value of the shares of the firm *i*, and the Long -Run LRMES MES, that is to say the capital loss when expected the market falls below 40% in six months.

Turbulence: measure proposed by Kritzman and Li (2010). It is a measure of excess volatility that compares the realized squared returns of financial institutions with their historical volatility:

$$Turbulence_t = (r_r - \mu)'\Sigma^{-1}(r_r - \mu)' \tag{A.12}$$

where r_t is the vector of returns of financial institutions, μ is the historical mean and Σ is the variance-covariance matrix.

Credit Default Yield Spread: spread between the yield of corporate bonds rated BAA and the rated AAA by Moody's.

TED Spread: difference between three-month LIBOR and three-month T-bill interest rates.

Term Spread: difference between yields on the ten year and the three month US Treasury bond. The series is obtained from Global Financial Data.